



# RESEARCH MEMORANDUM

A SURVEY OF UNCLASSIFIED AXIAL-FLOW -  
COMPRESSOR LITERATURE

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NATIONAL ADVISORY COMMITTEE  
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By Howard Z. Herzog and Arthur G. Hansen

## SUMMARY

A survey of unclassified axial-flow-compressor literature is presented in the form of brief reviews of the methods, results, and conclusions of selected reports. The reports are organized into several main categories with subdivisions, and frequent references are made within the individual reviews to pertinent material elsewhere in the survey.

## INTRODUCTION

This report, presenting a broad survey of the acquired information and the research progress into various aspects of the aerodynamics of axial-flow compressors, takes the form of brief reviews of the methods, results, and conclusions of selected reports on compressor investigations. The comments and evaluations in the reviews reflect statements by the authors of the reports being considered, unless specified otherwise.

The reports have been selected to obtain a comprehensive picture. Obviously, not all the related reports could be reviewed. Therefore, omission of a report does not necessarily reflect on its worth. Also, only the unclassified reports are reviewed, because much of the fundamental work in the major compressor problems is unclassified.

The entire survey comprises two main sections, the first reviewing the literature under the general heading of FLOW AND DESIGN THEORIES, and the second including mainly the experimental investigations into compressor performance and characteristics. Reports on end losses, secondary flows, works of general historical interest, and an alphabetical author index are also included. Many investigations involve analytical as well as experimental studies, so that the placing of a review within a theoretical or experimental heading is arbitrary. Assigning a report to a subdivision under a main heading is also arbitrary, because many reports touch upon several aspects of compressor research. The reports are placed on the basis of what was considered the main topic discussed in each report. Insofar as possible, references are made in the reviews in each subdivision to pertinent studies reviewed in other sections.

SECTION I. FLOW AND DESIGN THEORIES

## A. CASCADES, BLADE ROWS, STATORS, AND ROTORS

1. Ruden, P.: Investigation of Single-Stage Axial Fans. NACA TM 1062, 1944.

The axisymmetry assumption of an infinite number of thin blades gives the correct circumferentially averaged values of the fluid properties, provided the deviations of these properties from the average are small.

The assumption that all radial components vanish could not be maintained in estimating the behavior of an axial-flow fan over its entire operating range. Therefore, formulas for calculating stationary and rotating cascades are derived assuming small nonzero radial velocities. After individual airfoil-section calculations are made, a through-flow-distribution calculation is made which correlates operating conditions of the individual cascade sections. The Betz method (interference of neighboring profiles accounted for in a two-dimensional potential-flow solution about a central airfoil using the Betz chart) is extended by the use of a thickness correction, and diagrams are used to simplify the calculations. The through-flow calculation procedure leads to a differential equation which is reduced to a difference equation and solved by an iteration process.

2. Ferrari, Carlo: Un methodo rapido approssimato di calcolo delle caratteristiche aerodinamiche delle schiere di profili alari. Atti della Accad. delle Sci. di Torino, t. 81, no. 1, 1947, pp. 264-283. (A Rapid Approximate Method of Computing Aerodynamic Characteristics of Cascades of Airfoils.)

An approximate method of calculating the potential flow at small incidence angles past a cascade of thin low-cambered airfoils is based on a theory of slightly nonuniform flow past isolated airfoils.

3. Ferrari, Carlo: Sulla determinazione del profilo 'ottimo' per le pale dei compressori assiali. Rend. dell'Accad. Nazionale dei Lincei, t. 2, May 1947, pp. 576-586. (On the Determination of the 'Best' Profile of Blades for Axial Compressors.)

A transformation method is developed to determine the aerodynamic characteristics of thin low-cambered airfoils in frictionless flow and is restricted to compressors with small pressure rises. The criteria for the 'best' profile are based on requirements of uniform pressure distribution and flow-separation avoidance.

4. Goldstein, S.: Approximate Two-Dimensional Airfoil Theory. Pt. I - Velocity Distributions for Symmetrical Aerofoils. C.P. No. 68, British A.R.C. - Pt. II - Velocity Distributions for Cambered Aerofoils. C.P. No. 69, British A.R.C. - Goldstein, S., and Richards, E. J.: Pt. III - Approximate Designs of Symmetrical Airfoils for Specified Pressure Distributions. C.P. No. 70, British A.R.C. - Pt. IV - The Design of Centre Lines. C.P. No. 71, British A.R.C. - Pt. V - The Positions of Maximum Velocity and Theoretical  $C_L$  Ranges. C.P. No. 72, British A.R.C. - Goldstein, S., and Preston, J. F.: Pt. VI - Aerofoils with Hinged Flaps. C.P. No. 73, British A.R.C.

This series of reports presents a good development of both direct and inverse two-dimensional airfoil theories.

5. Richter, W.: Eindimensionale stationäre Gleichdruckströmung in bewegten Systemen. Ing.-Archiv, Bd. XVI, Heft 5/6, 1948, pp. 422-445. (One-Dimensional Stationary Flow at Constant Pressure in Moving Systems.)

One-dimensional compressible flow is investigated along a curve in a plane, a cylindrical surface, and a surface of revolution, with body force and constant static pressure. The plane can move with constant velocity in its own plane or can rotate about an axis perpendicular to the plane. The cylindrical surface can have motions parallel with its generator, and the surface of revolution can move along or rotate around its axis. All the problems are shown to reduce to linear motion in a plane; thus, the fluid paths (corresponding to "blade shape" in a compressor) can be calculated. Analytical and graphical solutions are given.

6. Richter, W.: Über eine Formel für Gleichdruckgebläse. Ing.-Archiv, Bd. XVII, Heft 1/2, 1949, pp. 88-93. (On a Formula for Axial Flow Compressors with Uniform Pressure.)

A one-dimensional analysis is presented of the flow through a compressor stage having constant static pressure through the rotor. The pressure is maintained constant by providing suitable decrease of the free cross-sectional area through the rotor. A relation is derived between the blade tangent- and cross-sectional area that relates blade curvature and curvature of the hub or outer casing and results in information on possible combinations of hub or outer-casing configurations for given tangential turning.

The one-dimensional theory neglects the influence of centrifugal forces on the flow due to curvature of the hub or casing, and also the effects of the tangential velocity components on radial distribution of velocity and static pressure.

This work follows survey 243.

7. Vazsonyi, Andrew: On the Aerodynamic Design of Axial-Flow Compressors and Turbines. Jour. Appl. Mech., vol. 15, no. 1, Mar. 1948, pp. 53-64.

A method for the theoretical flow of a perfect incompressible fluid through a cascade of arbitrary blades is presented. The lift coefficient can be determined as a function of angle of attack in 10 hours by a graphical procedure. By an extension of this method, the pressure and velocity distributions of a cascade of airfoils can be found for an entire range of solidities and staggers in approximately 60 man-hours.

8. Meyer, Rudolph: A General Method for the Computation of the Compressible Flow in Turbo Machines of Prescribed Boundaries and Blades. Nav. Res. Proj. 24748, Dept. Aero., U.S. Naval Postgraduate School (Annapolis), Feb. 1949.

A general method is developed for computing the isentropic steady compressible flow with subsonic relative velocities through stationary and rotating blade rows. Axisymmetric flow and thin blades are assumed, and the flow boundaries are coaxial surfaces of revolution. A graphical solution with use of successive approximations is presented.

The method, which is directed specifically toward solution of flows with large radial-velocity components, as in centrifugal compressors, is intended to analyze the flows within the passages, not just at the inlet and exit sections. Using "technically feasible" contours to find the flows (by use of meridional streamline curvatures) and indicating possible improvements of the blade shapes and contours are two of the objectives.

9. Monroe, Gerald Morgan: A Study of Compressible Perfect Fluid Motion in Turbo-machines with Infinitely Many Blades. Ph.D. Thesis, C.I.T., 1951.

Three-dimensional nonviscous but compressible flow past an infinite number of blades of arbitrary shape is studied. Large axial- and radial-velocity components are admitted to be possible. The infinitely many blades are represented by a "pseudo-conservative" force field, which is expressed as the product of two functions. One function expresses the rate at which energy is imparted to the fluid; the other is a potential function for the family of equations for the blade surfaces.

The idealized flow problem is formulated in terms of a stream function for velocities in the meridional plane, and a nonlinear differential equation results. The nonlinear action of rotational forces and compressibility effects is considered as a

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force displacing the stream surfaces from their position in irrotational incompressible flows. The character of the differential equations is determined by the relative velocities when the blades are present, and the meridional velocities when blades are not present. The solution is accomplished by use of relaxation procedures and an iteration process.

At transonic speeds a "cushioning effect" is found, where deflection of the streamlines is less than at either subsonic or supersonic speeds. This may explain in part the high efficiencies obtained for compressors with supersonic relative tip velocities.

10. Reissner, Hans: Blade Systems of Circular Arrangement in Steady, Compressible Flow. Studies and Essays. R. Courant Anniversary Vol., Interscience Pub., Inc., 1948, pp. 307-327.

A solution is presented to the complete inverse (blade-design) problem, in which no blade shapes or wall geometry are specified in advance. The method provides a first step in extending the solution from the infinite to the finite number of blades by use of a power series in the circumferential direction. The compression ratio, the form of the airfoils, and the flow around the blades can be determined if a prescribed variation of static pressure over every blade section and of the axial-velocity components along the machine axis is given.

The infinite number of blades (axisymmetric) solution is obtained first by quadratures for potential and nonpotential motion by use of Euler's continuity and energy equations. In the solution for finite number of blades with fixed spacing, it is first assumed that the streamline surface halfway between the blades is the same as in the axisymmetric case. Corrective terms for the nonsymmetric flow within the space between two adjacent blades are expanded in a power series in  $\psi$ , the angular distance from the streamline surfaces. Recurrence formulas for the corrective terms are derived. These corrective terms are added to the uniform flow obtained in the solution for infinite number of blades. The solution is simple if only those terms are kept which are linear in  $\psi$ , thus corresponding to linear pressure variation across the passage.

In the inverse problem for nonviscous flows, where the blade shape is to be determined, it is necessary to prescribe the components of the force field or energy distribution so that the force field will be perpendicular to the family of possible blade surfaces. This condition, as was pointed out much earlier by Bauersfeld, is necessary to ensure existence of the family of surfaces (cf. survey 98). However, this condition is not satisfied in the present paper.

11. Reissner, H. J., and Meyerhoff, L.: Analysis of an Axial Compressor Stage with Infinitesimal and Finite Blade Spacing. NACA TN 2493, 1951.

The analysis for axisymmetric infinite number of blades is extended to a solution for finite number of blades with fixed spacing, which reintroduces the derivatives with respect to the circumferential angle coordinate. The isentropic axisymmetric solution is taken as a first approximation. The effect of the blade system is replaced by a force field uniform in the circumferential direction, as is standard practice in this solution. The force field in turn is replaced for the finite-spacing case by inertia and pressure terms, which are correction terms for the flow variables in the axisymmetric solution.

In order to determine these correction terms, a series development is used for the velocity components and pressure functions in terms of powers of the spacing parameter  $\psi$ . One set of streamlines is assumed unchanged, that is, "frozen," and all the other streamlines must shift as the uniform force field is removed. The

changes in the flow variables, made by the correction terms, are assumed small enough that the powers higher than the first and products of these terms, along with their derivatives, may be neglected. Equations of first, second, and higher order are obtained for velocity and pressure variations, and solutions are effected by means of an iteration process.

12. Reissner, E. J., Meyerhoff, L., and Bloom, Martin: Two-Dimensional, Steady Non-viscous and Viscous Compressible Flow Through a System of Equidistant Blades. NACA TN 2718, 1952.

As in survey 11, the flow through a cascade of blades with fixed spacing is found. Here, however, the flow is merely assumed nonviscous instead of isentropic as in the previous report. The flow for the infinite number of close blades, in which the blade action is expressed by a continuous force field, must be found. Then, the force field between the blades must be replaced by inertia and pressure terms that had previously been omitted. A numerical example is given for  $90^\circ$  turning.

In an appendix, the viscous-flow case for blades with finite but narrow spacing is presented. This method is a generalization of the procedure for both nonviscous and isentropic flow. However, the boundary condition of closure of the blade surfaces at the leading and trailing edges is not satisfied.

13. Wu, Chung-Hua, and Brown, Curtis A.: A Theory of Direct and Inverse Problems of Compressible Flow Past Cascades of Arbitrary Airfoils. Jour. Aero. Sci., vol. 19, no. 3, Mar. 1952, pp. 183-196.

A unified approach to both the direct and inverse problems of compressible two-dimensional flow past a cascade of arbitrary airfoils is presented. The method is based on the correspondence in shapes between the blade mean line and the mean streamline in the channel and also on the observed close relation between variations in channel width and specific mass flow along the mean streamline.

The inlet and exit angles, blade thickness distribution, and either a desired blade mean line or the mean streamline shape are assumed known. The flow along the mean streamline is determined from these, and the solution is extended in the pitch direction by a Taylor series expansion obtained from the equations of continuity and motion.

In the inverse problem, the blade boundaries are determined by considering the mass flow at the inlet and interpreting the starting mean streamline as dividing the mass flow variously into two different portions. A number of profiles is thus obtained, and the profile with the best velocity distribution is chosen.

In the direct problem, a quick approximate solution following a similar expansion process is presented first. Successive corrections are applied for the shape of the flow path along the mean streamline until the desired velocity distribution is obtained. The process is described as quick and likely to be progressively speeded up as more families of airfoils are designed that provide additional background information. The method is recommended as a good first approximation for more accurate solutions by other longer methods. Examples calculated for turbine blades give favorable comparisons with experimental data.

14. Wu, Chung-Hua, and Brown, Curtis A.: Method of Analysis for Compressible Flow Past Arbitrary Turbomachine Blades on General Surfaces of Revolution. NACA TN 2407, 1951.

This one-dimensional analysis obtains a solution for the nonviscous compressible flow past turbomachine blades on a general surface of revolution. The analysis can

be used to obtain the shape of a mean streamline and the specific mass flow along it as a first step in a more complicated three-dimensional design procedure (see survey 103).

The solution begins with a stream surface of revolution known from an earlier analysis or an assumed shape. The equations of continuity and motion are combined into a nonlinear second-order differential equation in terms of a stream function defined for the flow. The differential equation is solved by difference methods. The use of fourth-degree Lagrangian polynomials and differentiation coefficients (survey 22) is recommended. The particular numerical procedures developed are used because the changes in the fluid properties passing through the turbomachine blades are, in general, large, and the shapes of the blades and surfaces are arbitrary. The methods of survey 22, which provide coefficients for differencing procedures for unequally spaced points, are used because of the curved boundaries. The large number of points needed make advisable the adoption of the higher-degree polynomial representation for large-scale digital-machine solution, if available. The methods of solution suggested and compared include solutions by a matrix method with large-scale digital-machine computation and a relaxation method for hand-operated desk-machine computations.

For numerical examples, the detailed flow variations in a highly cambered thick turbine blade configuration are obtained, with the following results: (1) The mean streamline shape approximates that of the mean channel line but has lower curvature. (2) Variation of the ratio of specific mass flow along the mean streamline to the inlet value follows roughly the variation of pitch to channel-width ratio. For the cases calculated, the blade curvature and thickness increased the specific mass flow along the mean streamline an average of 4 percent more than the area reduction due to blade-thickness effects. The influence of blade thickness extended a short distance upstream and downstream of the channel. (3) Variations in fluid properties across the channel can be represented fairly accurately by second-degree functions. (4) The velocity distributions obtained around the blade compare well with experimental data. (5) Use of the method enables evaluation of a correction factor for blade thickness (see survey 34).

15. Wu, Chung-Hua, and Brown, Curtis A.: A Method of Designing Turbomachine Blades with a Desirable Thickness Distribution for Compressible Flow Along an Arbitrary Stream Filament of Revolution. NACA TN 2455, 1951.

A rapid solution is given of the blade-profile design problem for steady two-dimensional compressible nonviscous subsonic potential flow, given the inlet and exit angles and certain geometric limitations, such as blade thickness. The report follows Betz, A., and Flügge-Lotz, I.: Design of Centrifugal Impeller Blades, NACA TM 902, 1939, and extends the solution of survey 14 to more general flow surfaces, taking into account the normal distances between the stream surfaces of revolution used.

As in survey 14, the velocity components and density are computed along an assumed mean streamline. The power-series expansion in the circumferential direction is made. Derivatives in the series are determined from the fluid state along the mean streamline from the continuity equation, the equations of motion, and the density-velocity relation for isentropic flow. Assuming different mean streamlines leads to different blade shapes. The rapid solutions were carried out by hand-operated desk computing machines in 16 hours. The speed of the method enables a choice of blade designs based, for example, on the most desirable velocity distributions on the blades obtained.



The more general nature of the flow surfaces permitted by this method enables use of the method for radial- and mixed-flow turbomachines, as well as for axial-flow cases. Some of the effects of three-dimensional flow can be accounted for in the design, because the variation in normal distance between the stream filaments of revolution is included in the method.

16. Wu, Chung-Hua, and Costilow, Eleanor L.: A Method of Solving the Direct and Inverse Problem of Supersonic Flow Along Arbitrary Stream Filaments of Revolution in Turbomachines. NACA TN 2492, 1951.

This method was developed to find, for supersonic relative velocities, (1) the shape of the mean stream surface dividing the mass flow between two blades into equal parts circumferentially, and (2) the correction factor  $b$  for blade thickness. These are used in conjunction with more elaborate flow theories for turbomachines (see survey 34). This method enables both the determination of the supersonic flow along stream surfaces of revolution for arbitrary blade shapes (the direct problem) and the design of such blades for prescribed velocity distribution or turning (the inverse problem).

The steady nonviscous supersonic flow along a stream filament of revolution of varying thickness is described by combining the continuity equation and the equation of motion in the circumferential direction in terms of a stream function. The derivatives are evaluated, and the flow equations are expressed in difference form along the two families of characteristic curves.

For the direct problem, the blade-to-blade flow variations for a given arbitrary supersonic stream filament of revolution are analyzed by successive calculations between the present calculation method and the through-flow solutions (see survey 34) until the solutions converge. The stream filament of revolution configuration is obtained from the through-flow solution. The shape of the mean stream surface and a blade thickness factor required for the through-flow calculations are provided to account for blade thickness and curvature effects.

Several illustrative examples are calculated. For the supersonic flows in nearly typical compressor configurations - that is, high solidity and thin blades - the mean streamline had lower curvature than the mean channel line (as in the subsonic case) but conformed closely to it. For thicker lower-solidity blades, the deviation of the mean streamline from the mean channel line was greater. The specific mass flow along the mean streamline increased on the average 9 percent more than anticipated from a one-dimensional analysis based on area reduction due to blade thickness for both thin and thick blades. This increase is more than twice the value obtained for the subsonic case (survey 14). In an analysis of symmetrical nozzles with no turning, the specific mass flow along the mean streamline for supersonic flow increased 8 percent over that anticipated from the one-dimensional analysis.

17. Wu, Chung-Hua, Brown, Curtis A., and Prian, Vasily D.: An Approximate Method of Determining the Subsonic Flow in an Arbitrary Stream Filament of Revolution Cut by Arbitrary Turbomachine Blades. NACA TN 2702, 1952.

As a first step in the more general three-dimensional compressor design theory (survey 103), a quick approximate solution for the subsonic nonviscous flow past arbitrary turbomachine blades on arbitrary surfaces of revolution is presented. It is an extension of the two-dimensional solution of survey 15 to arbitrary surfaces of revolution. The first part of the calculation proceeds exactly as in survey 15; that is, the fluid properties in the circumferential direction are obtained by a Taylor series expansion. At this point, with the blade shapes given, the mass flows between the two blade surfaces and the chosen streamline can be computed. The relative

percentages of mass flow on either side of the streamline are compared with known inlet conditions at various positions along the streamline, and the assumed shape of the streamline and the specific mass flow along it are adjusted. This process was completed in about 16 hours in several numerical examples, and the results compared favorably with other numerical and experimental results.

18. Krajewski, Bohdan: Analiza przepływu w kanałach sprężarek osiowych wielostopniowych. Tech. Lotnicza, vol. 7, no. 2, Mar./Apr. 1952, pp. 44-47. (Analysis of Flow in Ducts of Multistage Axial Compressors.)

Kármán's method is described and applied to calculations of shapes and dimensions of ducts in multistage axial-flow compressors.

19. Spannhake, W.: Vortex Theory of Flow Through Turbomachines. Pt. I - Infinite Number of Blades. Final Rep., Office Naval Res., Washington (D.C.). (Contract Nonr 322(00), Proj. No. 6008 and Illinois Inst. Tech.)

The axisymmetric theory of flow through compressors with an infinite number of thin close blades is given using the bound vortex concept and is extended to include compressible flows. Both analytical and graphical methods of solution are indicated, and illustrative examples are discussed.

20. Betz, A.: Energieumsetzung elastischer Gase in Schaufelgittern. Forsch. Geb. Ing.-Wes., Bd. 18, Nr. 3, 1952, pp. 61-71. (Energy Conversion of Compressible Gases in Cascades.)

A simple graphical method is presented for calculating the changes in fluid properties for compressible flow through straight or circular cascades. Curves for use in the solutions are presented for cases of  $\gamma = 1.4$  and  $\gamma = 1.3$ . From these curves, limiting flow conditions are readily apparent. The method includes, approximately, the effects of nonparallel walls, blade thickness and motion, fluid losses, and internal temperature rise due to viscous action.

21. Schwaar, P.: Quelques Remarques sur Le Calcul Aero-Thermodynamique de L'Aubage des Turbomachines Axiales. Bull. Tech. de la Suisse Romande, t. 78, no. 19, Sept. 20, 1952, pp. 245-251. (Some Remarks on the Aerothermodynamic Calculation of Blades in Axial Turbomachines.)

In the later stages of a compressor the actual velocity profiles are likely to be quite different from the assumed velocity profiles, thus resulting in excessive losses. More complete information is required about the fluid coming from a stator or a rotor for accurate blade-shape design for such conditions. A calculation method for axisymmetric flow based on thermodynamic relations is provided to account for radial distributions of energy and efficiency and for the radial displacement of the streamlines.

The solution is obtained by an iteration process which is laborious but amenable to considerable simplification if the first trial shows the radial displacement to be small.

According to the author, the polytropic efficiency of turbines is lower than for compressors because the typical turbine design neglects radial displacement.

22. Wu, Chung-Hua: Formulas and Tables of Coefficients for Numerical Differentiation with Function Values Given at Unequally Spaced Points and Application to Solution of Partial Differential Equations. NACA TN 2214, 1950.

Analyses of the flows through typical turbomachine configurations often lead to relations expressed in partial differential equations. These equations may then be expressed in difference form, and the computations proceed numerically, for example, by relaxation methods. The presence of the curved boundaries in the turbomachine configurations makes it important to find the derivatives of the functions near the boundaries in terms of the values of the functions at unequally spaced intervals. In the present report, such general differentiation formulas are obtained for the successive derivatives of a function in terms of the values of the function at unequally spaced intervals. Lagrange's interpolation formula, with error terms, is used for the function at  $n$  points to obtain expressions for the successive derivatives. In typical flow problems, the grid near a curved boundary may have unequally spaced points at only one end of the interval used. For these commonly occurring cases, tables of coefficients for the first four derivatives (for cases of  $n = 3, 4, 5$ ) in the formulas are given for intervals of 0.01, and for different ratios of this end spacing to the others, varying from 0.1 to 1.29.

The formulas and coefficients obtained can be used to give approximate values of the various derivatives at any point within a range of given arguments when the values of the function are given at a number of points unequally spaced. Thus, the formulas and coefficients are useful for the numerical integration of the partial-differential equations where, for example, the starting value is given only on a curve unequally spaced from a regular grid line. They would also prove useful in changing an interval size during calculations. In the numerical solution of an elliptic partial-differential equation, these formulas and coefficients can be employed to express the finite-difference terms for very small intervals more accurately and conveniently at points near a curved boundary, and to allow for increasing grid size away from the boundary. In this fashion, these coefficients help make the solution of such problems practical on large-scale digital computers.

The formulas and coefficients are applied to the compressible flow past isolated and cascade airfoils as an example.

#### B. SINGLE-STAGE AND MULTISTAGE COMPRESSORS

23. Traupel, Walter (C. W. Smith, trans.): New General Theory of Multistage Axial Flow Turbomachines. Navships 250-445-1, Navy Dept., Washington (D.C.).

This report is an early exposition of the axisymmetric-flow theory in turbomachines. The first five chapters present a complete workable axisymmetric design theory for axial-flow compressors and turbines, and the last chapter considers more general problems.

The nonviscous flow theory is presented for infinite number of blades with constant energy addition radially and with cylindrical walls. Radial force and other momentum terms are neglected. The measurements required for the calculation of stage performance and characteristics and coefficients for steady flow in the main stream are also discussed. Flow near the walls is not considered. Numerical examples are given. The problem of calculating stage performance and characteristics on the basis of cascade tests is studied. The discussion considers the errors involved in calculating the work output and efficiency of a stage on the basis of cascade tests as well as the errors involved in the assumption of nonviscous flow. Loss coefficients are developed and related to efficiency, which is expressed as a function of the drag-lift ratio, clearance losses, and wall losses. The design calculations are extended to multistage turbomachines, and numerical examples are provided. The three-dimensional effects, which arise as a result of radial variation in blade shape, from inclined blades, and from compressible flow, are considered.

24. Pabst, O.: Application of the Calculation Methods Used for Single Stage Axial Blowers to Multi-Stage Blowers. G.D.V. Rep. No. 4, RTP/TIB Trans. No. GDC 16/18T, British M.O.S.

The single-stage theory of survey 1 is applied to multistage units. The occurrence of radial velocities, the relations between stage and multistage efficiency, and the decrease in total efficiency as the pressure ratio increases are discussed. The optimum value is found when the pressure rise is the same in rotor and stator.

25. Pabst, O.: Theoretische Vorarbeiten für die Entwicklung von Achsialgeblasen mit hoher Druckziffer. GDV-BR-5, Focke-Wulf Flugzeugbau G.m.b.H., Aug. 1941. (Theoretical Preliminary Work for the Development of High Pressure Axial Compressors. Trans. By Beech Aircraft Corp., 1947.)

Equations are derived for pressure coefficients with efficiency and rotor-flow stability as criteria. The calculations indicate Mach number effects on the obtainable pressure rises. Axial-flow compressor design is affected by three main factors: (1) Pressure rises are limited by the flow stability mainly because of the hub design conditions and the effects of stator reaction. (2) Rotor efficiency considerations limit pressure increases. (3) Compressibility effects manifest themselves through the choice required of pressure coefficient and flow velocities.

26. Eckert, Bruno, and Group: Minutes of Axial Flow Compressor Meeting of 26 June on German Articles. Code 445A, Navy Dept., Bur. Ships, Washington (D.C.), 1946.

Among the items covered are characteristic numbers for compressor performance, nomenclature, losses, design bases, choice of stage pressure coefficients, influence of blade shape and camber, blade stagger and shape required for the velocity diagrams, use of cascade tests and single-stage tests for determining the preceding, stage matching, supersonic compressors, and correlations between the performance and design features of various German axial-flow compressors. A summary of survey 23 and Weinig's flow theory are presented.

27. Wattendorf, Frank L.: Theory and Design of Axial Flow Fans or Compressors. Tech. Rep. No. 5155, Army Air Forces, Wright Field, Dayton (Ohio), Oct. 16, 1944. (Notes taken by Hugh E. Gommel.)

A good review of blade-element theory and design development for perfect flow is presented, and axial-flow compressor problems and methods for increasing compressor performance are discussed in detail.

Blade-element theory is compared for compressible and incompressible flows for low-solidity blade rows. Losses and loss estimates are discussed in terms of such factors as blade losses and profile drag, tip clearance, duct entrance losses, diffusion, and residual rotation. Stage pressure-rise and Mach number limitations are considered, and the following means for increasing stage pressure rise are investigated: increasing the lift coefficient by means, for example, of boundary-layer control, slots, flaps, and so forth, and using higher speeds, higher inlet Mach numbers, and closer spaced blades.

The second portion of the report deals with higher-solidity cascades, including higher values of chord to spacing than are possible to study by elementary airfoil analysis methods. The relations between pressure and mass-flow requirements of a system are discussed in terms of radial variations of pressure and flow coefficients, power characteristics, blade-shape design, surge points, compressibility effects, and choking limits.

28. Wattendorf, Frank L.: High-Speed Flow Through Cambered Rotating Grids. Jour. Aero. Sci., vol. 15, no. 4, Apr. 1948, pp. 243-247.

High-speed adiabatic compressible flow through a cascade of rotating blades is analyzed in a one-dimensional approximation using average channel values. The overall performance is investigated for several cases: (1) subsonic flow everywhere, (2) supersonic flow in downstream stator, (3) supersonic flow in rotor and stator but subsonic axial flow, and (4) supersonic flow throughout.

The results of the analysis indicate that large compression ratios are theoretically possible. By the use of highly cambered blades, the stage pressure ratio can be increased considerably, but probably at the expense of reduced efficiencies.

29. Wattendorf, Frank L.: Axial Compressors - Simplified Design Comparisons with Regard to High Mass Flow. AF Tech. Rep. No. 6042, U.S. Air Force, Air Materiel Command, Wright-Patterson Air Force Base Dayton (Ohio), Sept. 1950. (See also Jour. Aero. Sci., vol. 18, no. 7, July 1951, pp. 447-459.)

The investigation arrives at design conditions permitting attainment of high mass flows. Four design types are compared: (1) free-vortex distribution with positive and negative prerotation, (2) rigid body velocity distribution with positive prerotation, (3) maximum constant rotor work with no prerotation specifications, and (4) nonconstant work radially. The inlet stage is the only stage considered, because it limits mass flow at design condition. The hub-tip radius ratio and axial Mach number effects on the mass flow and temperature rise, with assumed limitations on the relative and absolute Mach numbers are discussed, as well as the relation of the stage pressure ratio and temperature rise for the various designs.

The following results are obtained: (1) Free-vortex flow with positive prerotation design is not suitable for high mass flow and stage pressure ratio; (2) free-vortex flow distribution with negative prerotation is suitable for high mass flows but results in low stage pressure ratios; (3) rigid body velocity distribution with positive prerotation yields higher pressure ratios than the free-vortex design for the usual mass-flow range; and (4) the radially nonconstant work design is capable of providing high stage pressure ratios up to extremely high mass-flow rates.

30. Davis, Hunt: A New Method for the Aerodynamic Design of Multistage Axial-Flow Compressors. Jour. Aero. Sci., vol. 15, no. 1, Jan. 1948, pp. 41-48.

A graphical method described herein for the determination of blade angles for an axial-flow compressor of free-vortex design utilizes the same rotor and stator forms for several successive stages by merely changing the spacing and the root stagger angle. The optimum inlet Mach number for the first stages of an axial-flow compressor of free-vortex design is derived and is related to the given inlet conditions, dimensions, and tip speeds.

31. Howell, A. R.: Fluid Dynamics of Axial Compressors. War Emergency Issue No. 12 pub. by Inst. Mech. Eng. (London), 1945. (Reprinted in U.S. by A.S.M.E., Jan. 1947, pp. 441-452.)

A simple two-dimensional-fluid dynamic theory for low-speed flows in compressor cascades is based on two-dimensional wind-tunnel tests. Correction factors are applied for losses and for mean stage conditions in compressors. The discussion includes compression, efficiency, flow characteristics, and blade-element performance. A compressor-performance analysis and a numerical example of stage performance are given. The three-dimensional effects that occur in the two-dimensional-cascade tunnel are shown.

32. Emmons, Howard W., and Ball, George A.: The Present Status of Axial Flow Compressor Design. NAW-3662, Harvard Univ., 1947.

The actual flow through an axial-compressor is not obtainable from theoretical solutions, although theories may predict the flows fairly well at the design point. The theories generally fail to predict the performance of a new design over the entire performance range accurately. The report discusses the complexities of the flow through an axial-flow compressor and gives the results of certain investigations and a chart of references.

Four main topics are included in the first section. The first, ideal-fluid effects, considers various aspects of compressibility effects and radial flows and their causes. The second, boundary-layer effects, discusses briefly the various loss coefficients used in design, the limited applicability of single airfoil and cascade tests, and the need for more detailed boundary-layer data from compressors themselves. The third main subject is secondary flow, which is caused by blade tip leakage, relative motion between blade tips and casing, flow turning, and radial flow in blade boundary layers. The fourth real-flow complication discussed concerns blade-row interaction effects, in particular, the timewise variation of the flow encountered by a rotor blade downstream of a stator row. Flow velocity varies in both magnitude and direction because of the stator-wake effects.

33. Wu, Chung-Hua, and Wolfenstein, Lincoln: Application of Radial-Equilibrium Condition to Axial-Flow Compressor and Turbine Design. NACA Rep. 955, 1950. (Supersedes NACA TN 1795.)

In this first of a series of reports dealing with increasingly inclusive and complex design procedures, the general equations for three-dimensional flow in axial-flow turbomachines are derived in terms of the velocity, total enthalpy, and entropy from the equation of state, the Navier-Stokes equations of motion for a real fluid, the energy equation, and the continuity equation. These equations are combined into a series of general flow equations which are reduced for the steady-flow axisymmetric case by neglecting viscosity and by assuming all partial derivatives of the gas properties, with respect to the circumferential direction and time, to be zero. The common assumption of simplified radial equilibrium is not made in this report.

The analysis indicates that six independent relations are available among eight independent variables. The designer thus has two degrees of freedom at his disposal; that is, he can specify a desirable variation of two of the gas properties. This is done, generally, at stations between the blade rows. Several types of compressors that can be obtained by different ways of using these degrees of freedom are discussed.

The assumption of axisymmetric flow averages out circumferential variations of the fluid properties; thus, by not assuming simplified radial equilibrium, the solutions obtained emphasize the effects of radial motion of the gases and the radial distribution of the gas properties through the turbomachine. No analytical solutions are presented, but several numerical methods of solution are suggested. One method, in particular, assumes a sinusoidal radial-flow path between stations.

The results of this analysis indicate that the radial motion depends upon the blade aspect ratio, passage taper, hub-tip radius ratio, main flow velocity, and velocity diagram. The analysis gives useful information concerning the flow within the blade rows themselves, rather than the flows outside the blade rows.

34. Wu, Chung-Hua: General Through-Flow Theory of Fluid Flow with Subsonic or Supersonic Velocity in Turbomachines of Arbitrary Hub and Casing Shapes. NACA TN 2302, 1951.

The general through-flow theory is an extension of survey 33. The line of investigation can be traced from survey 23, concerning the incompressible simplified-radial-equilibrium, axisymmetric-flow solution, to survey 33, concerning the compressible nonsimplified-radial-equilibrium flow solution, to survey 10, which presents a blade design solution for compressible flow with arbitrary hub and casing walls and finite spacing, to the present report. In this report a unified theory, for both the direct and inverse problems, is developed for compressible subsonic or supersonic velocities with arbitrary hub and wall shapes with approximate correction factors for blade-thickness effects.

The through-flow theory considers the flow along an arbitrary surface. A principal equation for this flow is developed from the continuity equation and the equation of motion in the radial direction, and by use of a stream function. (Retaining the blade force terms in the equations of motion partially accounts for the circumferential variations of pressure.) Two main groups of designs are considered: (1) free vortex, wheel, symmetrical-velocity diagram types, and others, and (2) radial-blade-element configurations such as high-speed centrifugal and mixed-flow impellers, and others. Numerical methods are presented for solving the principal equation for both the elliptic and hyperbolic cases, corresponding roughly to subsonic and supersonic flows, respectively.

An approximate blade-thickness factor can be included in the definition of the stream function. Analyses of the blade-thickness effects on flow along the mean streamline are presented for subsonic compressible and incompressible flows in survey 14 and for supersonic flows in survey 16.

The through-flow solution (survey 34) can be considered as a limiting case for a more general three-dimensional solution, such as is presented in survey 103. It can be regarded as the solution for the flow along a mean stream surface (called "S<sub>2</sub> surface" in survey 103) dividing the mass flow between adjacent blades equally, when the circumferential variation of the velocity components approaches zero. Thus, the through-flow solution indicates trends when the effects of having a finite number of blades are small or constant. In this connection, the use of a blade-thickness factor, based on information about blade-thickness effects as obtained in surveys 14 and 16, should improve the solutions considerably.

35. Wu, Chung-Hua, Brown, Curtis A., and Costilow, Eleanor L.: Analysis of Flow in a Subsonic Mixed-Flow Impeller. NACA TN 2749, 1952.

A through-flow analysis (survey 34) is made of three flow conditions through a high-solidity mixed-flow impeller. The results are used to evaluate the usefulness of the general method of through-flow analysis for turbomachines as well as the usefulness of incompressible solutions. To do so, both the incompressible and the compressible nonviscous-fluid flows through the impeller are analyzed. The results of the analysis are presented as contour plots of a stream function, the velocity components, total enthalpy, static pressure, and Mach number.

The results indicate that the general through-flow method can be useful in analyzing the mean flow through a turbomachine. The trend of flow variation was found to be similar for compressible and incompressible flow; therefore, the general trend of compressible-flow variations can be determined from an incompressible solution. The variation trend of the theoretically determined static pressures along the casing agrees well with available experimental data.

36. Wu, Chung-Hua: Subsonic Flow of Air Through a Single-Stage and a Seven-Stage Compressor. NACA TN 2961, 1953.

An axisymmetric-flow analysis method which assumes a sinusoidal radial streamline flow path through a turbomachine blade row is developed in survey 33. This analysis determines the effects of radial flow displacement in addition to the effects of flow-path curvature radially on the radial distribution of air state downstream of a blade row. The through-flow analysis (survey 34) solves the flow for high-solidity thin blades and accounts for blade thickness effects. Survey 103 extends the analysis to a three-dimensional solution for a finite number of thick blades. The usefulness of the method of assuming simple sinusoidal radial-flow paths is evaluated by comparisons with results obtained from the through-flow and three-dimensional analyses. The solutions are made for the steady nonviscous compressible flow for both low- and high-speed subsonic flow through a single- and a seven-stage compressor designed for a symmetrical velocity diagram at all radii. The solutions were obtained by relaxation procedures, and some were checked further by matrix methods.

The axial-velocity distributions obtained by the more elaborate solutions agreed well with the simpler approximate solution assuming sinusoidal radial-flow paths. The analysis resulted in other conclusions, as follows: (1) The radially increasing angular momentum of the fluid particles for this design type and the curved hub and wall contours have a predominating influence on the flow distribution. Sizable radial flows are generated. (2) Compressibility does not greatly change the shapes of the streamlines but does affect the velocity components. (3) A small amount of radial twist in this type of blading has a negligible effect on the flow distribution. (4) The air in this case has a generally inward radial movement through the guide vanes and the first rotor. The movement is outward in the first stator. (5) This oscillatory radial flow throughout the compressor has a period equal to the length of the stage. (6) The oscillatory radial flows are largest in the inlet stages. Assuming simple sinusoidal radial-flow paths is superior to ordinary simplified-radial-equilibrium solutions (see survey 104).

37. Bowen, John T., Sabersky, Rolf H., and Rannie, W. Duncan: Theoretical and Experimental Investigations of Axial Flow Compressors, Pt. I. Mech. Eng. Lab., C.I.T., Jan. 1949. (Navy Contract N6-ORI-102, Task Order IV.)

This report, which is the first in a series (surveys 167, 201, and 65) of investigations of axial-flow turbomachines, develops a new theory of perfect fluid flow in axial turbomachines in a form useful for design purposes. This axisymmetric flow theory, in which the exit-flow angle is used as a basic parameter, uses a new linearizing method to simplify the flow equations. Methods are suggested for taking advantage of unconventional blading in order to obtain increases in flow rate over that with conventional blading.

Axisymmetric flow and cylindrical or nearly cylindrical bounding walls are assumed, and radial force terms and other momentum terms are neglected. Equations are derived for the distribution of axial velocity through the stators and rotors at design and off-design conditions.

In this approach, the streamlines are designed to shift radially toward the hub as they near the rotor. Therefore, the relative velocity decreases at the rotor tip and increases at the hub, thus reducing both the tip Mach numbers and the deflection angle at the hub. Consequently, the rotor tip speed can be raised or the hub-tip radius ratio can be decreased to give a larger annulus area. It is estimated that an over-all 40- to 50-percent increase in flow rate can be achieved in this way. However, the flows can no longer be considered by means of two-dimensional theories.



Vortex blading for two-dimensional flows is compared with solid-body rotation and other types of blading to determine the effects of radial variation of circulation. Even for vortex-type blading, off-design conditions lead to large changes in axial-velocity distribution.

An experimental compressor was designed and built according to this theory but for very low flow Mach numbers and tip speeds. The absolute value of the exit angle agrees experimentally with Constant's rule (survey 134). Boundary-layer calculations for loss on two-dimensional cascades are made and compared with losses measured on similar sections in the compressor-stator row. The relation between inlet and exit angles is almost identical for sections of the compressor blading and experimental data taken from corresponding two-dimensional cascades, except near stall. High losses are noted in the end-wall boundary-layer region. The blade profiles should be adapted to the wall boundary layer.

In general, tests on the experimental compressor designed by the method indicate close agreement between theory and experiment at low flow rates and low tip speeds.

38. Gazarin, Adel: Graphische Behandlung der kompressiblen und inkompressiblen Strömung durch Turbomaschinenstufen. Zurich, 1951. (Graphical Treatment of Compressible and Incompressible Flow Through Turbomachine Stages.)

A graphical method for solving the system of equations for axisymmetric isentropic flow between blade rows is presented. This method uses a modified radial-equilibrium condition that ignores the curvature of the meridional streamlines. The radial variations of velocity and density in the gap between blade rows are obtained, from which the stage characteristics and the shapes and locations of the meridional streamlines are determined. These streamlines are found to have periodic wave shapes, with period equal to length of stage.

Four kinds of stages are examined in detail: (1) constant mass flow per unit area in planes between blade rows and behind the stage, (2) untwisted blades, (3) blades twisted for solid-body flow, and (4) constant reaction radially. Remarks on the influence of viscosity, secondary flows, radial boundary-layer flow, tip-clearance flow, and blade geometry are given.

39. Geimer, Robert G., and Cardinal, Saviour, Jr.: Development of a System of Simultaneous Equations for Axial-Flow Compressor Design Based on Ideal Vortex Theory. B.A.E. Thesis, Rensselaer Polytechnic Inst., 1951.

A derivation of a system of simultaneous equations is proposed as a basis for a simplified method of axial-flow-compressor design by ideal vortex theory. The simultaneous equations are classified into two groups: one to use in the design of the inlet guide vanes and the first stage, the second to design the other stages. No solution is obtained because of the complexity of the equations, but a procedure for a first approximation is suggested.

40. Seyler, D. R., and Wilson, C. B.: A Design of a Single Stage, Low Speed, Axial-Flow Compressor. B.A.E. Thesis, Rensselaer Polytechnic Inst., 1951.

Design and performance of a low-speed single-stage axial-flow compressor are analyzed, based on free-vortex flow theory. The axial velocity was kept constant and the blading was designed by airfoil theory. The compressor was designed for the purpose of investigating the secondary flows resulting from shed vortices.

41. Van Allen, Richard I.: A Study of an Analytical System for the Design of an Axial-Flow Compressor Using Simultaneous Equations. B.A.E. Thesis, Rensselaer Polytechnic Inst., 1952.

Following survey 39 the results of a system for the design of an axial-flow compressor by means of sets of simultaneous equations are presented. These equations take into account the effects of friction and compressibility on the flow through the compressor. That the flow paths of the gas can be maintained as predicted by the equations, by altering the blade angle settings in such fashion as to overcome the effects of losses, is a fundamental assumption in the design system. Several methods for solution of the equations are given.

42. Jarre, Gianni: Compressori assiali a vortice libero. Monografie Sci. di Aero., Nr. 12, Sept. 1952. (Axial-Flow Compressors with Free Vortex.)

The motion of the fluid in a multistage compressor is studied in order to establish the relations among the velocity field of the fluid, the force field imposed by the blading, and the physical characteristics of the fluid. The rigorous derivation of the aerodynamic equations assumes nonviscous axisymmetric flow. The conditions characterizing free-vortex axial-flow compressors are deduced. A mathematical and numerical investigation of this family of compressors is given, and degrees of action and reaction, and effects of viscosity, compressibility, and mechanical loads are examined.

The design of the first stages, the characteristics of multistage and periodic compressors (compressors in which the flow at each stage exit has velocities equal in magnitude and direction to those at the compressor inlet), the conditions for maximum efficiency and performance, and the aerodynamic characteristics of the fluid are investigated. Numerical examples are provided.

43. Smith, L. H., Jr.: The Design of a Subsonic Multistage Axial-Flow Compressor for Maximum Power Per Stage. M.S. Thesis, The Johns Hopkins Univ., 1951.

Design equations for the steady compressible nonviscous axisymmetric flow of a fluid through a multistage compressor are derived. A general discussion of the factors limiting the power output of a stage is followed by an analysis determining velocity distributions and diagrams for a multistage compressor that will maximize the power output of the first stage (for incompressible flow here) for a given tip diameter and maximum flow Mach number relative to the blades. Prerotation and non-uniform energy addition radially are found necessary. (Relative efficiencies of various possible types of blade loading are not considered here.) More energy is added nearer the tip than the hub in the first stage in order to keep the axial-velocity distribution reasonable. This can be corrected in subsequent stages so that the final discharge flow is more nearly uniform.

The solutions thus far constitute a first approximation. Graphical techniques are employed for a second approximation agreeing closely with the first despite the presence of large energy gradients and highly rotational flow. While losses due to rotationality of the fluid are not likely to be severe, the large fluid turning angles encountered in the later stages of this design, together with sizable velocity decreases, may lead to flow separation.

44. Traugott, S. C.: A Two-Stage Axial-Flow Compressor with Nonuniform Energy Addition Per Stage. M.S. Thesis, The Johns Hopkins Univ., 1951.

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The compressor problem of maximizing mass flow or rotor pressure, or both, is formulated. Considerations of restricting local relative velocity levels to the subsonic range and maintaining radial equilibrium lead to a design with nonuniform energy addition. The author's purpose is to express and solve the equations describing the two-dimensional approximations to the three-dimensional flow in forms suitable for design.

The flow is assumed steady and nonviscous, with negligible compressibility effects. The curvature of the streamlines is considered negligible at the stations of the investigation, and the radial distance between the streamlines is considered constant. While the flow is thus assumed on cylindrical stream surfaces upstream and downstream of the blade rows, the radii of these surfaces of revolution are not necessarily equal. The equations for the flow are developed and applied to the design of a two-stage compressor having prerotation other than the usual free-vortex or wheel types and nonuniform energy addition in the rotors, in an effort to increase the compressor output above standard design. Two independent parameters are to be chosen initially; the prerotation and blade tip speed are used.

The conclusions reached are as follows: High pressure ratios with high mass flows can be achieved for compressors by considering the variations of axial velocity. For a two-stage compressor good through-flow was actually achieved at high pressure ratios. For the particular design under discussion, an increase in tip speed would result in less through-flow, larger blade-turning angles, and greater variation of axial velocity from hub to tip but with some increase in power output. These results, obtained by a graphical solution, are not general. The effects of streamline curvature in the radial plane are indicated.

45. Smith, L. H., Jr., Traugott, S. C., and Wislicenus, G. F.: A Practical Solution of a Three-Dimensional Flow Problem of Axial-Flow Turbomachinery. Trans. A.S.M.E., vol. 75, no. 5, July 1953, pp. 789-803.

A design method is presented for obtaining a compressor which, with given Mach number limitations, will produce the maximum delivery pressure with maximum mass-flow rate possible. The method actually determines the meridional flow patterns through turbomachines with nearly free-vortex flow for axisymmetric frictionless flow. The computational procedure is also presented.

The development of the theory provides a clear physical picture of the internal workings of an axial-flow turbomachine, in the opinion of the reviewers, and serves as the forerunner of a series of investigations (e.g., surveys 235 and 236).

46. Novak, R. A.: Notes on the Fundamentals of the Design of Multi-Stage Axial Compressors. Lecture No. 6, Gas Turbine Lectures, Dept. Mech. and Ind. Eng., Univ. Mich., June 29-July 10, 1953.

A broad general discussion of the over-all design of axial-flow compressors, touching upon nearly all aspects of the field with the exception of secondary flows, is presented. An attempt is made to reduce complicated derivations to a minimum and to present a clear physical picture that will enable the designer to use his judgment skillfully where necessary. The main topics considered include ideal two-dimensional-flow theories, blade-element theory, equilibrium conditions, sample vector-diagram calculations, determination of the geometry and performance of a cascade theoretically and semi-empirically, Mach number and Reynolds number effects, effects of design variables and operating conditions on performance, rotating stall, blade end clearance, multistage-compressor performance, stage-stacking techniques, and over-all design procedure for a multistage compressor.

## C. EFFECTS OF STREAMLINE CURVATURE AND RADIAL VARIATION OF CIRCULATION

47. Eckert and Korbacher: The Flow Through Axial Turbine Stages of Large Radial Length. NACA TM 1118, 1947.

The analysis concerns uniform, nonviscous, axisymmetric flow with zero whirl at the inlet on cylindrical surfaces. Simple radial equilibrium is likewise assumed. The flow through the blading is affected by spanwise variations of pressure, important for long blades. A theoretical method is developed for determining the effects of the pressure variation and centrifugal effects due to the whirl component of the flow in both twisted and untwisted rotor and stator blading. Experimentally obtained results on untwisted blades with  $30^\circ$  and  $45^\circ$  discharge angles confirm the calculations. The calculations of vorticity distribution in the fluid lead to a discussion of trailing-edge vortices shed from blades with a non-free-vortex circulation distribution radially. The flow deflects toward the axis in nontwisted stator blades and deflects toward the outer casing in nontwisted rotor blades.

48. Tsien, Hsue-Shen: Loss in Compressor or Turbine Due to Twisted Blades. CIE Jour., 1947, pp. 40-53.

Radial flows in and behind a rotating wheel result from centrifugal forces and radial pressure gradients and can have undesirable effects on the blade boundary layers. To avoid these difficulties, a twist is sometimes designed into the moving blades or into the flow leaving the guide vanes. The resulting nonuniform force and circulation distributions along the blades lead to induced velocities, which rotate the flow around the blades and change the effective angle of attack. Thus, it is impossible to calculate the actual lift from the geometrical angle of attack. The result is reduced correspondence between the actual and design performance, associated reduced pressure rises, and so forth.

A method for calculating these effects is presented. A general blade equation is developed for the relation between the geometric configuration and the aerodynamic behavior of low-cambered blades of nearly constant chord, spacing, and lift-curve slope radially for two-dimensional nonviscous flow. An extension to nonuniform flow is presented in survey 237.

49. Marble, Frank E.: The Flow of a Perfect Fluid Through an Axial-Flow Turbomachine with Prescribed Blade Loading. Jour. Aero. Sci., vol. 15, no. 8, Aug. 1948, pp. 473-485.

The mathematical aspects of the problems associated with spanwise variations in circulation are stated. Even for nonviscous incompressible axisymmetric flow, the flow solution is simple only for the vortex distribution of the tangential velocity component, that is, constant circulation spanwise. For this condition, there are no radial or axial disturbances, and the flow can be considered two dimensional in the sense that the streamlines all lie on cylindrical surfaces. When there is varying circulation spanwise, the streamlines no longer follow cylindrical surfaces. According to Prandtl's theory, vortices trail from the blades with their vorticity vectors pointing downstream in the flow direction. The equations for this flow are now nonlinear nonhomogeneous partial-differential equations. Nevertheless, it is still possible to calculate the difference between the axial-velocity profiles far upstream and far downstream by neglecting the radial transport of vorticity, or by assuming the flow at those stations has negligible radial components (cylindrical walls, see survey 37). The problem of the velocity distributions near the blade rows is considered (complementary to survey 33, which considers the flow paths within the blade rows). The differential-flow equation is linearized by assuming that there is no self-transport of the vorticity, which is transported only by the mean velocity.

This is equivalent to considering disturbances in the radial- and axial-velocity components small compared with the mean axial velocity. It is further assumed there is no inner- or outer-shroud taper (cf. survey 10, which solves the complete inverse problem of arbitrary walls and blades but allows no vorticity to be shed). The solutions are found for the radial-, tangential-, and axial-velocity components induced by a single rotating or stationary blade row with finite chord and any prescribed blade loading. The solutions to this linearized equation provide an approximation to the velocity field for any blade-loading distribution, and are likely to be useful for moderate-turning high-solidity configurations, that is, when self-transport of vorticity may be negligible.

When the blade chord approaches zero (actuator-disk theory) the linearized partial-differential equation becomes homogeneous and can be solved much more readily and directly using the given boundary conditions. This is equivalent to assuming a discontinuity in the tangential-velocity component at some axial location. Suitably choosing this location enables this direct solution to be a reasonable approximation to the more nearly exact and more arduous solution for the case of nonzero blade chord. The direct solution does not provide a reasonable approximation for operation near the lifting line.

In both the zero- and nonzero-chord cases, the linearized-flow equations make it possible to obtain solutions for multistage turbomachines by superposition of individual blade-row solutions. Simple methods of approximating the flow through a turbomachine are developed from exponential approximations for the velocity component distributions. These results were quick enough and accurate enough to be useful in estimating interference effects of adjacent blade rows.

An illustrative example is calculated for a hub-tip radius ratio of 0.6, aspect ratio of 2.0, and increasing circulation from hub to tip. Disturbances associated with stationary blades attain their maximum downstream of the blade midchord position. Those associated with rotating blade rows attain their maximum upstream of the blade midchord position.

50. Marble, Frank E.: Some Problems Concerning the Three-Dimensional Flow in Axial Turbomachines. Preprint No. 182, Inst. Aero. Sci., 1949.

The rapid exponential approximations to the linearized-flow solution developed by survey 49 are applied to the investigation of the streamline-curvature effects (caused by radial motion) on the radial distribution of the fluid near a stationary or rotating blade row. The manner of transition from the far upstream to the far downstream velocity profile is examined. A solution is obtained for the entire flow field for a compressor with cylindrical walls, solid-body rotation out of the inlet guide vanes, and similar successive stages with constant energy addition radially in each. The radial adjustment ranges from nearly complete at the trailing edges of blades of aspect ratio 1.0 to almost negligible at the trailing edges of blades of aspect ratio 3.0.

These results are applied to two important compressor problems: (1) the mutual interference of neighboring blade rows in multistage axial-flow machines, and (2) the three-dimensional flow effects at off-design operating conditions. For blade aspect ratios near 1.0, interference effects between adjacent blade rows are negligible, and distortion of the axial-velocity profile at off-design operation is small. The effects of off-design operation may increase considerably with increasing blade aspect ratio. At aspect ratios over 3.0, the interference effects may become pre-dominant flow factors.

51. Legendre, Robert: Remarques Diverses sur L'Encoulement dans les Turbomachines a Circulation Variable. Assoc. Tech. Maritime et Aeronautique (Paris), 1950. (Some Remarks on the Flow in Turbomachines with Variable Circulation.)

Considerable theoretical difficulties associated with calculating the flow through an axial-flow compressor with circulation varying radially are discussed (see survey 49). A suggestion is made for an approach using Prandtl's theory for circulation about an airfoil of finite aspect ratio.

52. Pinnes, R. W.: The Theoretical Possibilities for Balanced Flow in Compressor and Turbine Design. Paper No. 50-A-66, A.S.M.E., 1950.

Large amounts of blade twist are required in free-vortex-design compressors. This constitutes a serious manufacturing and stress problem. The Mach number limitations are likewise restrictive. Other velocity distributions are investigated in an effort to achieve peak efficiencies under practical operating conditions.

The theoretical investigation concerns nonviscous axisymmetric isentropic flows with no radial velocity or curvature and with constant total energy radially. A family of generalized balanced flows is investigated for which the whirl velocity is considered to vary as some power of the radius. The study of the theoretical range of operation of these flows reveals that only free-vortex flow can be maintained over an entire operating range. The other flows can be maintained only under certain limiting conditions when it is desired to obtain equal work radially and yet avoid critical ranges. For example, solid-body rotation has a limited range. Starting with solid-body rotation at the rotor inlet and extracting equal work radially results in nonsolid-body rotation at the rotor exit. The succeeding stators then have to restore the flow to solid distribution for the next rotor inlet. This process soon produces critical velocities.

A "Balanced Flow Chart" is constructed which shows the range of balanced flows theoretically possible for any given set of operating conditions in axial-flow machinery.

#### D. TIME-UNSTEADY FLOW EFFECTS

53. Keller, C.: Kinetic Energy Losses Behind Blade Grids as a Result of Periodic Variation in the Circulation. Rep. Inst. Aero., Tech. Hochschule (Zurich), 1934, pp. 167-187. (Available as R.T.P. Trans. No. 1883, British, M.A.P.)

The kinetic energy transferred to the vortex wake due to periodic variations in circulation is calculated for several airfoil configurations. These calculations assume two-dimensional potential flows and assumed the variation in circulation to be sinusoidal in amplitude with time. The results indicate that the kinetic energy transferred to the wakes is found proportional to the square of the amplitude of the circulation variation. This result is obtained for isolated airfoils, for airfoils in cascade staggered as well as unstaggered, and for rotating blades in lightly loaded turbine stages.

54. von Kármán, Theodore, and Tsien, Hsue-Shen: Lifting-Line Theory for a Wing in Non-Uniform Flow. Quart. Appl. Math., vol. III, no. 1, Apr. 1945, pp. 1-11.

A general linearized theory is developed for a wing subjected to nonuniform inlet flow. General expressions of the induced velocity and drag are presented that are applicable to compressor blades with little camber.

55. von Kármán, Th., and Sears, W. R.: Airfoil Theory for Non-Uniform Motion. Jour. Aero. Sci., vol. 5, no. 10, Aug. 1938, pp. 379-390.

The fundamental concepts of the two-dimensional circulation theory of airfoils are presented. Timewise variation of the circulation around an airfoil is shown to produce a "wake of vorticity," for which induced vorticity distributions over a thin airfoil are presented. Formulas are obtained for the lift and moment of the thin airfoil with timewise-variable circulation. The lift is seen to consist of the sum of three parts: (1) Quasi-steady lift  $L_0$ , representing the lift that would be produced if the instantaneous velocity and angle of attack were steady; (2)  $L_1$ , representing the forces acting on a body in an ideal fluid when no circulation is produced because of the reaction of the accelerated fluid masses; and (3)  $L_2$ , representing the contributions to the lift from the wake itself. A corresponding formulation is obtained for the moment.

These general results are employed to study an oscillating airfoil and an airfoil entering a vertical gust. The lift always acts at the quarter-chord point of the airfoil.

56. Sears, William R.: Some Aspects of Non-Stationary Airfoil Theory and Its Practical Application. Jour. Aero. Sci., vol. 8, no. 3, Jan. 1941, pp. 104-108.

The general expressions for lift and moment developed in survey 55 are applied to a thin airfoil in a series of alternating up and down gusts sinusoidally distributed. The magnitude of the lift decreases as the frequency of the gusts increases. A wind-tunnel fan blade operating downstream of a set of vanes is investigated. The vanes produce regular disturbances through which the fan blades pass periodically, giving rise to periodical forces and, thus, perhaps to dangerous vibrations. An approximate calculation, however, revealed no dangerously large vibration amplitudes.

57. Kemp, Nelson H., and Sears, W. R.: Aerodynamic Interference Between Moving Blade Rows. Jour. Aero. Sci., vol. 20, no. 9, Sept. 1953, pp. 585-597.

The results for an airfoil moving through a stationary gust pattern obtained in survey 56 are extended to provide formulas for calculating the time-unsteady effects for a single stator row followed by a single rotor row. The incompressible non-viscous two-dimensional flow past a cascade of thin slightly-cambered lightly loaded blades is studied. Each airfoil is considered to be in a velocity field induced by (1) its own wake, (2) the variable bound vortices of other blades in its own blade row, (3) their wakes, (4) the bound vortices of blades in the other row, and (5) their wakes. A scheme of successive approximations is set up. However, any calculations beyond the first would be extremely difficult.

Numerical investigations were made for the first approximation on configurations of thin low-turning airfoils with rotor and stator solidities of 1.0, with a 50-percent reaction stage, and with 45° stagger angles. The calculations showed large increases of the unsteady lift as the gap between the blade rows narrowed. The ratio of the pitch of rotor blades to the pitch of the stator blades affected the unsteady lift appreciably. For one case, the unsteady lift was found to be 18 percent of the steady lift. The power in the vortex wakes, expressed as a ratio of the energy transferred into the wakes per unit time to the steady power required to turn the rotor, was small.

The energy in the stator wake for the case calculated was about 100 times as large as the energy in the rotor wake. According to the results of survey 53, the unsteady circulation on the stator blades is about 10 times that on the rotor blades.

58. Kemp, Nelson H., and Sears, W. R.: Aerodynamic Interference Between Moving Blade Rows. Graduate School Aero. Eng., Cornell Univ., Dec. 1, 1952. (Contract AF 33(038)-21406.)

Much the same material appears here as in survey 57. A different numerical example is calculated in which the chordwise-velocity distribution on the stator and rotor are quite different. The induced circulation on a blade row due to the relative motion of an adjacent row downstream is greater than that due to an adjacent row upstream.

59. Abe, Shun-ichi: On the Mutual Interference Between the Impeller and the Guide Vane of Axial Flow Pumps or Fans. Rep. of Inst. High Speed Mech., Sendai (Japan), vol. 4, no. 38, Mar. 1951.

Theoretical calculations are presented on the effects of mutual interference. The flow through the impeller and guide vanes is assumed to be similar to the potential flow between equivalent streets of vortex sheets, one moving and the other stationary. The equivalent street of vortex sheets for a blade row is chosen on the basis of having the same values of zero-lift angle, coefficient of lift, and moment coefficient as the corresponding blade elements.

The results of the calculations indicate that the lift on the blades varies periodically because of mutual interference. The amplitudes of the variations increase as the gap between the rows decreases (see survey 57).

60. Bullock, Robert O., Wilcox, Ward W., and Moses, Jason J.: Experimental and Theoretical Studies of Surging in Continuous-Flow Compressors. NACA Rep. 861, 1946. (Supersedes NACA TN 1213.)

Surging, a periodic-pulsating flow occurring when the slope of the characteristic curve is positive, limits the operating range of compressors at low flow rates. Surging has been investigated extensively in an effort to determine its causes and control in order to extend the surge-free operating range of compressors.

An experimental investigation was conducted on a centrifugal compressor using a differential-pressure recorder to estimate the magnitude and frequency of surging pressure pulsations as functions of the volume of the compressor system and the impeller tip speed. An analysis of surging in an assumed simplified configuration indicates how the flow instability may arise in a compressor and how it may be inhibited.

In the experimental studies on a first test unit, the outlet throttle was gradually closed until surging occurred. Inlet and outlet total-pressure traces were recorded during the process as were the inlet and outlet velocity pressures (differential between total and static pressure) and the outlet static pressure. In reversing the procedure, it was necessary to open the outlet throttle beyond the point of the surging onset in order to stop it (see survey 61, root-to-tip stall). This overlap, called a hysteresis effect, was quite large under certain conditions.

In a second unit, surging occurred at low volume flows as in the first test, and also at two points of relatively high volume flow (see survey 61, progressive propagating stall). The pulsations at the higher flow values were not so violent as those limiting the low-flow range of the second test. In each case, however, the slope of the characteristics curve was positive when surging occurred.

The third unit, which was constructed to enclose a minimum volume, used nine different combinations of inlet and outlet conditions. The lowest frequency and most violent pressure fluctuations occurred when the external pipes enclosed the largest volume. This result agrees with previous work by other investigators.



The simplified analysis presents an expression for a useful stability criterion  $S$  in terms of the inlet acoustical velocity, inlet effective area, length and volume, and the slope of both the throttle and compressor characteristic curves. On the basis of this formulation, a physical interpretation of surging is developed. Compressibility effects increase the tendency toward unstable operation.

Several methods of inhibiting surge, such as variable-angle prerotation vanes, flow recirculation, and decreasing the capacity of the external piping in relation to the compressor capacity are discussed.

61. Huppert, Merle C., and Benser, William A.: Some Stall and Surge Phenomena in Axial-Flow Compressors. Jour. Aero. Sci., vol. 20, no. 12, Dec. 1953, pp. 835-845.

The authors are concerned mainly with defining carefully the problems of stall and surge in compressors. Stall and surge are recognized as operation-limiting phenomena at low-flow values and low speeds (see survey 60) that limit the engine acceleration rates, the operational flexibility in the use of adjustable jet nozzles or other thrust-augmentation devices at intermediate compressor speeds, and the operation of an engine at high thrust for greater-than-design equivalent speeds. The stall of a multistage compressor results from stall of one or more stages. Low-speed stall is associated with flow breakdown in the inlet stages. At speeds near design values, stall of a multistage compressor is associated with difficulties encountered in the rear stages.

The use of high-frequency-response instrumentation such as hot-wire anemometers and pressure transducers show stage stall to be an unsteady-flow phenomenon. Zones or sectors of the annulus have very low mass flows through them compared with the remainder of the annulus. These zones of low flow rotate circumferentially about the compressor axis in the same direction as the rotor, but at lower speeds. The number of the stall zones was not predictable on the basis of compressor geometry.

Two types of single-stage stall are recognized. One is progressive stage stall, in which multiple stall zones originate at the blade ends and extend over a portion of the blade span. These stall zones can be obtained, as before, by closing the outlet throttle to reduce the flow rate. As the flow is reduced beyond the first appearance of the propagating stalls, the individual stall zones enlarge. Sudden increases in stall-zone numbers may occur with continued flow reduction, and the spanwise extent of the stalls may also increase. The behavior of progressive rotating stall leads to a gradual drop in the pressure coefficient.

The other type of stall encountered is root-to-tip stall in which the stall zone extends over the entire blade span, usually in stages with high hub-tip radius ratio. Only one root-to-tip stall at a time was observed. The onset of root-to-tip stall results in a sharp drop in pressure ratio. The hysteresis effect noted by survey 60 was found here as well.

Stages with low hub-tip radius ratio with progressive stalls are associated with stages with high hub-tip radius ratio with root-to-tip stalls. Intermediate stages first develop progressive stalls, which change to root-to-tip stalls accompanied by sharp pressure drops and hysteresis effects.

Single-stage stall is necessary but not sufficient for over-all multistage-compressor stall. Progressive propagating-stall patterns originating in the early stages may extend axially almost through the entire compressor and still not drastically affect the pressure coefficient of the compressor if the intermediate

and rear stages are operating well above their stall values. A stage-stacking analysis is developed enabling computation of the operating characteristics of a multi-stage compressor from knowledge of the individual stage characteristics. This stage-stacking analysis is applied to a hypothetical compressor. The study was able to demonstrate which stage was responsible for the flow discontinuity due to root-to-tip stall at three rotor speeds. At the low speeds the discontinuities result from disturbances in the inlet stages, and at high speeds the rear stages are the sources of the disturbances. Interactions between individual stage effects may further deteriorate the over-all compressor performance characteristics. For example, progressive stall in the early stages may sufficiently disturb the over-all flow picture sufficiently to cause early root-to-tip stall in the intermediate stages. Stage interactions could very well lead to root-to-tip stall in the inlet stages, causing kinks in the over-all compressor stall-limit line. These phenomena definitely limit the range of compressor operation, whether or not they induce compressor surge.

Two types of compressor surge are shown to exist: (1) the classical type (see survey 60) due to classical system instabilities, and (2) the limit-cycle surge due to discontinuities in the pressure-ratio flow characteristic of the compressor. The authors are careful to point out the differences in surge and stall characteristics between compressors in jet engines and in typical test configurations.

62. Emmons, H. W., Pearson, C. E., and Grant, H. P.: Compressor Surge and Stall Propagation. Trans. A.S.M.E., vol. 77, no. 4, May 1955, pp. 455-467.

The importance of understanding and controlling surge in relation to the problems of starting and rapidly accelerating a jet engine have led to this hot-wire-anemometer investigation. Two distinct surge phenomena in the low flow range are reported. One is a self-excited Helmholtz organ-pipe resonance, and the other is a blade-row flow instability, self-propagating from blade to blade. At low speeds, these two kinds of disturbances are separated by a stable range of operation (see survey 60). At higher speeds, they spread progressively, until, at high enough speeds, they overlap.

The general mechanism of the propagating-stall type of disturbance is described as in survey 61. Basic equations are developed, relating the rate of pressure change with mass flow leading to an expression for pulsation frequency. The over-all analysis relates the propagation rate and stall behavior to loss effects in the blade row, expressed in terms of an effective flow area coefficient  $\alpha$ , which is proportional to the ratio of an effective flow area (sufficient to pass the exit weight flows with no losses present) to the exit area. The propagation rate is thus related to boundary-layer growth parameters and thereby is found proportional to wheel speed. (See survey 66 for a discussion of the assumptions involved here.)

63. Sears, W. R.: On Asymmetric Flow in an Axial-Flow Compressor Stage. Jour. Appl. Mech., vol. 20, no. 3, Sept. 1953, pp. 442-443.

Asymmetric flows can occur in compressor configurations when the blades are operating close to stall conditions and a time lag occurs in developing lift. (See survey 65, on the suitability of this mechanism for propagating a disturbance proposed here.) The conditions necessary for asymmetric flow are analyzed theoretically. For a steady uniform stream with an infinite number of blades of large radius and small chord (i.e., actuator-disk analysis), the rotational flow effects due to vortices shed by the blades are important for asymmetric flows. The velocities induced by the asymmetric loading are calculated. Moreover, it is found that these induced effects can support the circulation distribution that gives rise to them.

A simple relation established between the circumferential circulation distribution and the induced velocities at the disk is evaluated for assumed sinusoidal distributions.

64. Sears, W. R.: A Theory of "Rotating Stall" in Axial-Flow Compressors. Graduate School Aero. Eng., Cornell Univ., Ithaca (N.Y.). (Contract AF 33(038)-21406.)

This theoretical investigation is an extension and generalization of survey 63. The airfoil approach of the earlier work is adopted here, but the restriction to small inlet swirl relative to the rotating flow pattern is removed. Assuming the proper time lag required to establish a separated boundary layer, steadily rotating asymmetric flow patterns can be obtained which rotate in the same direction as the inlet swirl, but at lower speeds than do the blades. The theory indicates the possibilities of the pattern rotation opposite to the direction of blade rotation. No solutions are obtained for zero time lag.

A second approach incorporates the viscous profile drag effects by assuming two empirical channel relations based on experimental observations, namely, that the discharge flow angle and the dimensionless total-pressure rise are functions only of the entering angle. For this approach, asymmetric rotating flow patterns can exist with or without time lag.

While these steadily rotating self-supporting rotating-stall patterns exist only in special circumstances of blade geometry and aerodynamics, little advice can be given in how to avoid these circumstances.

65. Iura, T., and Rannie, W. D.: Observations of Propagating Stall in Axial-Flow Compressors. Rep. No. 4, Mech. Eng. Lab., C.I.T., Apr. 1953. (Navy Contract N6-ORI-102, Task Order 4.)

A series of experimental investigations of stall phenomena were made in a three-stage free-vortex blading compressor, a one-stage free-vortex blading compressor, and a one-stage solid-body blading compressor. The results of careful measurements of stalling in these compressors are reported. Two principal types of stall, partial and full, were observed (corresponding to progressive and root-to-tip stall, respectively, in survey 61).

Partial-stall (progressive-stall) characteristics are as follows: (1) One or more stalled regions may be present. The stalled regions extend over only part of the blade span but cover more than one blade passage and may be at either blade hub or tip, depending on the configuration. (2) Transition into partial-stall conditions occurs when the outlet throttle is closed beyond the maximum exit pressure. Continued closing of the throttle (decrease of flow coefficient) results in either increasing the number of stall regions propagating at the same speed as before, regrouping of regions into a larger partial-stall regions propagating at a lower speed (see survey 67, in which speed increases with stall region size), or regrouping into a single full-stall (root-to-tip) region. Partial stall with continued decrease of flow coefficient causes a steady gradual decline in pressure. Transition to full stall is accompanied by a sudden sharp drop in exit pressure. (3) Partial stall occurs at higher flow coefficients than full stall, propagates at higher speed, and has a maximum ratio of velocity-fluctuation amplitude to mean velocity of 0.6 to 0.65 as compared with about 0.7 for full stall.

Full-stall (root-to-tip stall) characteristics are as follows: (1) Only one stalled region is present, and it extends over the full blade span and covers more than one passage. (2) Transition into full stall occurs as the flow coefficient is decreased (by closing exit throttle, e.g.) beyond a certain point. Continued decrease

in flow coefficient enlarges the full stall (retarded-flow) region. The propagating-stall speed varies with solidity but not with a decrease in flow coefficient. To relieve the full-stall condition by opening the throttle, it is necessary to open the throttle beyond the point where full stall first occurred (i.e., hysteresis effect is noted). Transition to full stall is characterized by a sudden sharp drop in exit pressure. (3) Full-stall propagating speed is lower than partial-stall propagating speed (but in same direction) and occurs at lower flow coefficients. At a given flow coefficient the full-stall region is widest at the same radial position where partial stall is widest for that particular configuration.

The discussion of the experimental results points out that the characteristic time bases for estimating propagating-stall speed by various investigators (surveys 66 and 63) are believed to depend upon the time required for the establishment of a separated boundary layer when an airfoil is subjected to a sudden change in angle of attack. The comparative constancy of full-stall propagating speed over a wide flow range, obtained experimentally in this report, does not support this view. A characteristic time, depending upon the inertia of the flow rather than upon the detailed nature of the flow, is suggested in survey 67.

The actual total-pressure loss due to stall in a compressor cascade was small and nearly constant for a range of incidence angles of  $8^\circ$  to  $10^\circ$ . Outside this range, the total-pressure losses rise rapidly. This total-pressure-loss variation is the most markedly nonlinear flow characteristic observed, and the amplitude of the stall fluctuations that appeared seemed to be typical of nonlinear oscillations. Therefore, the total-pressure-loss variation is an important factor in determining the stall-fluctuation amplitudes.

It was not positively established whether the rotor or stator was primarily responsible for stall. Large aerodynamic blade excitations can arise as a result of stall, and it is possible that blade failures could result from partial stall (more likely than at full-stall conditions). The effects of stalls extend far upstream and downstream of the blade row of their occurrence. A discussion of stalling and the characteristic hysteresis loops involved in going from stalled to unstalled and back to stalled conditions leads to a tentative suggestion that surging may involve an oscillation between two stall patterns.

66. Marble, Frank E.: Propagation of Stall in a Compressor Blade Row. Tech. Rep. No. 4, GAICTT, Jan. 1954. (Office Sci. Res., Air Res. and Dev. Command Contract AF 18(600)-178.)

Stall propagation in a blade row using a two-dimensional small-perturbation theory and assuming no interference from adjacent blade rows is discussed. Blade spacing and blade chord are not critical quantities determining the stall propagation, and therefore the two-dimensional cascade is further simplified by replacing it with an actuator line. Given the inlet flow angle and the local turning angle for the unstalled cascade, the local static-pressure rise and outlet angle are known in the absence of losses. Thus, for given turning, assigning a different static-pressure rise at a given inlet angle is equivalent to assigning a particular total-pressure loss at the value of inlet angle. In a simplified model, the static-pressure rise is assumed zero in a stalled region. The results of the theoretical analysis of this model indicate that the propagation rate of the stalled region is determined by the relative inlet flow angle to the cascade and is independent of the flow turning. From a plot of the relative speed of stall propagation against inlet angle, it is observed that the propagation speed in this simple cascade configuration is least sensitive to variations of relative inlet flow angles near the value  $\pi/4$ . As this is a common value of inlet flow angle, this result for the simplified case agrees well with observed behavior of the relative constancy of propagating-stall speed as

a compressor is throttled. The most influential factor determining the extent of the stalled region is the cascade static-pressure rise.

For a somewhat less simplified case, in which the pressure rise and discharge angle are now dependent upon the inlet angle near stall (respectively zero and independent in the first simplified case), the speed of propagation is found to be a function of the inlet and discharge angles. However, the inlet flow angle is still the predominant factor. The cascade static-pressure rise is still the important factor determining the extent of the stalled region.

The effects of propagating stall in the performance of a single compressor stage are also discussed. The compressor performance is evaluated in terms of flow and pressure coefficients. Specifying these coefficients is equivalent to specifying the relative inlet and discharge angles in conformity with the procedures in the rest of the report. One result obtained is that the pressure loss due to stall is proportional to the stalled fraction of the circumference. The performance of a hypothetical compressor is computed, and qualitative agreement with experimentally observed behavior is obtained.

67. Stenning, Alan H.: Stall Propagation in a Cascade of Airfoils. Jour. Aero. Sci., vol. 21, no. 10, Oct. 1954, pp. 711-713.

The mechanism in cascade flow that controls stall-propagation speed and the size of stall regions is investigated. Three factors are considered; the inertia of the fluid both outside the cascade and inside the passage, and the response time for the establishment of a separated boundary layer when subjected to change in angle of attack. A linearized one-dimensional perturbation analysis leads to several interesting conclusions. The speed of stall propagation increases with the size of the stall region (cf. observations in survey 65) and depends mainly on the inertia characteristics of the fluid approaching the cascade (the pressure changes downstream of a cascade are much smaller than upstream) and between the blades. The boundary-layer response time factor decreases the speed of propagation but its effect is not as large as that of the over-all inertia. The boundary-layer analysis indicates that stall-region size and propagation depend on the inlet angle (see survey 61).

68. Smith, A. G., and Fletcher, P. J.: Observations on the Surging of Various Low-Speed Fans and Compressors. Memo. No. M.219, British N.G.T.E., July 1954.

A series of observations of stall and surge phenomena in centrifugal- and axial-flow compressors is reported and analyzed briefly. Results and conclusions of investigations using wool tufts and pitot pressure tubes are presented. Only those findings that are different from the results of investigations of surveys 61 to 67 are recorded here.

Only one band of rotating reverse-flow region, which rotates at a speed of 0.30 to 0.50 of rotor speed in the majority of cases, is noted. In one case, a type of surge was observed which involved uniform backflow at the rotor tips all around the periphery. By considering a compressor as multiple compressors in parallel, a hysteresis effect and operating characteristic similar to observed findings can be predicted. A tentative theory for the rotating surge discussed here leads to calculations of propagation speeds based on inertia effects of the fluid in the diffuser channels. According to this theory, surge frequency should decrease as the number and length of the channels (solidity) goes up. This theory does not apply to axial-flow fans, but it is speculated that there the propagation rate would probably decrease with smaller pressure drops due to the surging.

69. Sears, William R., and Chu, Wen-Hwa: Some Aerodynamic Problems of Compressors and Turbines. Lecture Ser. No. 30, Inst. for Fluid Dynamics and Appl. Mech., Univ. Maryland, Nov. 1953.

Three major internal-flow problems of turbomachinery are analyzed: (1) interference between moving blade rows, (2) progressive rotating stall as a consistent solution of symmetric flows, and (3) airfoil characteristics after stall.

An analysis of interference effects for a two-dimensional nonviscous incompressible flow is developed as an iterative process, following the work of survey 57. One result is that the unsteady forces can be as large as 15 percent of the steady lift, which may lead to blade vibration problems. The circulation induced on a given blade row by the relative motion of a downstream row is greater than that induced by an upstream row.

Two-dimensional actuator-line theory was applied to an analysis of rotating stall. The flow upstream of the actuator line was irrotational. Results indicate that special combinations of blade characteristics and geometry must exist in order for steady rotating patterns to be present.

#### E. COMPRESSIBILITY AND MACH NUMBER EFFECTS

70. Perl, W., and Epstein, H. T.: Some Effects of Compressibility on the Flow through Fans and Turbines. NACA Rep. 842, 1946. (Supersedes NACA ARR E5G03.)

A theoretical analysis applies the laws of conservation of mass, momentum, and energy to a study of compressible nonviscous flow through a two-dimensional cascade of airfoils. A fundamental relation is derived between the upstream and downstream flow angles  $\lambda_1$  and  $\lambda_2$ , respectively, inlet Mach number  $M_1$ , and pressure ratio  $p_2/p_1$  across the cascade:

$$\frac{\cos \lambda_1}{\cos \lambda_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \sqrt{1 - \frac{2}{(\gamma-1)M_1^2} \left[ \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}$$

This result is presented graphically in a series of curves and compared with the incompressible-flow relations to demonstrate the compressibility effects for compressor or turbine cascades. The plots reveal two ranges of flow angles and an inlet Mach number for which no ideal pressure ratio exists; in certain other flow-angle ranges two possible pressure ratios are predicted for given inlet Mach numbers.

The effects of variable axial-flow area are treated. In the range of Mach numbers less than 0.4 in axial-flow compressors, the relations for compressible and incompressible flow both give nearly the same results. Some implications of the basic conservation laws are discussed for nonideal flow through cascades.

71. Traupel, W.: Kompressible Strömung durch Turbinen. Schweizer Archiv f. Angewandte Wiss. und Tech., Jahrg. 16, Heft 5, May 1950, pp. 129-138; Jahrg. 16, Nr. 6, June 1950, pp. 176-186. (Compressible Flow Through Turbines.)

A theory is developed for the mathematical analysis of frictionless isentropic flow through axial-flow turbines and compressors when compressibility effects are large. The radial-equilibrium condition is analyzed, and it is concluded that the radial distribution of density determines the magnitude of compressibility effects.

72. Lieblein, Seymour, and Sandercock, Donald M.: Compressibility Correction for Turning Angles of Axial-Flow Inlet Guide Vanes. NACA TN 2215, 1950.

The Prandtl-Glauert compressibility correction for isolated airfoils is extended to accelerating cascades with axial air inlet and moderate blade cambers and solidity. The total lift coefficient of the cascade is considered as the sum of two parts: (1) the coefficient of lift associated with constant airspeed across the cascade, and (2) the coefficient of lift associated with the change of speed across the cascade. As a result, the variations of lift coefficient and turning angle with inlet Mach number at a given angle of attack are obtained analytically for two-dimensional flow. The results are then adapted for annular flows.

Some experimental data are presented to demonstrate the agreement achieved. The turning angles derived from the two-dimensional flow analysis are corrected to constant axial velocity across the cascade; consequently, the theoretical variation of corrected turning angles up to  $29^\circ$  and for axial-velocity increases up to 20 percent across the vanes is in agreement with experimental turning-angle data from investigations of several axial-flow-compressor inlet-guide-vane designs in annular cascades. Thus, generalized blade-element compressibility corrections are presented for use in the design and analysis of axial-flow-compressor guide vanes.

73. Sinnette, John T., Jr., Costello, George R., and Cummings, Robert L.: Expressions for Measuring the Accuracy of Approximate Solutions to Compressible Flow Through Cascades of Blades with Examples of Use. NACA TN 2501, 1951.

Four error expressions are derived as integral relations based on the condition of irrotationality and on the laws of conservation of mass and momentum. These integral expressions, necessary conditions for steady irrotational compressible flow through a straight cascade of blades, are (1) the ratio of the difference between upstream and downstream mass flow to upstream mass flow, (2) the ratios of the differences between momentum changes in two directions (calculated from blade surface pressure distributions and upstream velocity vectors) to resultant blade force, and (3) the ratio of the circulation around a simply connected region to circulation around an airfoil. The error expressions are applied to tests for the compatibility and accuracy of three different types of approximate solutions for compressible flows through the cascade. These approximations are incompressible potential solutions, the Prandtl-Glauert approximation as adapted to cascades, and an approximation based on an assumed linear pressure-volume relation as applied to cascades. Pressures and densities are also obtained for comparison from the exact Bernoulli equation and pressure-density relation.

The compatibility expressions are used to compare the preceding approximate solutions and to measure the over-all degree of approximation achieved by each. The results indicate that the accuracy of the incompressible potential-flow approximation decreases with rising Mach numbers. The inaccuracies involved in the process of obtaining the incompressible solution are insignificant compared with the inaccuracy of the incompressible approximation itself for Mach numbers of the order of 0.8.

The Prandtl-Glauert approximation results in errors of about the same magnitude as the incompressible approximation when used with a linearized velocity relation. Much improved answers result when the exact velocity relation is used.

The linear pressure-volume-relation approximation gives much more accurate results than either the Prandtl-Glauert or the incompressible potential-flow approximations. The errors of approximation apparently are only slightly greater than the errors involved in the calculations, such as rounding off numbers, integration approximations, and so forth.

## F. VISCOUS FLOWS, BOUNDARY LAYERS, AND LOSSES

74. Wu, Chung-Hua: Survey of Available Information on Internal Flow Losses Through Axial Turbomachines. NACA RM E50J13, 1950.

The available information on internal-flow losses in axial-flow compressors and turbines is briefly surveyed, and both the present state of knowledge and the direction of probable future research on reduction of these losses are discussed.

75. Squire, H. B., and Young, A. D.: The Calculation of the Profile Drag of Aerofoils. R & M No. 1838, British A.R.C., Nov. 1937.

The drag on an isolated airfoil in two-dimensional incompressible flow is calculated for a range of Reynolds numbers for several airfoil thicknesses on the basis of the pressure distributions about the airfoil computed by means of potential-flow analyses.

The boundary layer on the airfoil is assumed to be laminar from the leading edge back to an assumed transition point and is considered turbulent rearward from that point. In the laminar layer, the skin friction and boundary-layer thickness near the leading edge are computed by Pohlhausen's method. The turbulent boundary-layer properties are computed using the Prandtl theory together with the experimental results of Nikuyadse and Buri. For the wake, it is assumed that the momentum thickness at the trailing edge is equal to the sum of the momentum thicknesses of the boundary layers of the upper and lower surfaces near the trailing edge. The static pressure is assumed constant in the wake to permit the use of the boundary-layer equations. Comparison of calculated results with experiment is fairly satisfactory.

76. Weske, J. R.: Drag of Airfoils in Grids of High Solidity. Jour. Aero. Sci., vol. 11, no. 4, Oct. 1944, pp. 369-372.

The drag on NACA 0012 symmetrical airfoils is analyzed for a cascade at zero stagger and zero lift. The results indicate that the profile drag is composed of two component parts: (1) the drag produced by the normal pressures (e.g., the displacement of the main flow by the boundary layer), which was found to be small at high Reynolds numbers, and (2) the drag produced by skin friction. Causes of profile drag which arise in the wakes are of particular concern. The drag coefficient is computed in terms of momentum thickness as in survey 75. For these airfoils, the transition point in the grids is located accurately at the point of maximum thickness of the airfoils. Data are obtained by an experimental investigation to enable integration of the momentum equation.

Three major conclusions are reached: (1) The drag of high-solidity airfoils can be calculated fairly well from the momentum equation for a turbulent boundary layer by assuming a constant mean value for a coefficient of skin friction. (2) The momentum transfer of the wake in a field of adverse static-pressure gradients gives rise to losses much larger than for individual airfoils. This effect is significant only for very high solidities. (3) Displacement of the flow was observed from the blade ends toward the midspan, especially at high solidities (called jet contraction elsewhere) and produces separation at the blade ends while the flow at the midspan adheres. At high solidities, the result is considered to increase the drag beyond that predicted solely on the basis of profile drag.

77. Loitsianskii, L. G.: Resistance of Cascade of Airfoils in a Gas Stream at Subsonic Velocity. NACA TM 1303, 1951.



The drag of airfoils in cascade is theoretically calculated as interactions among the boundary layers of the individual blades. This method is based on a slight flow nonhomogeneity in the section of the aerodynamic wake where the boundary layers of the individual blades merge. To compute this method, exact expressions in terms of total load loss, circulation, and free-stream velocity are set up for the reactions on the cascade in viscous incompressible flow. The assumption that the velocity is nearly uniform at the downstream station where the wakes merge leads to an expression for total-pressure loss and reaction normal to the cascade axis as functions of the boundary-layer momentum thickness at that station. This result is then related to the momentum thickness at the blade trailing edges.

Thus, for a given pressure distribution on the profile, the momentum thickness and an estimate of the cascade reaction can be made.

78. Schlichting, H., and Scholz, N.: Über die theoretische Berechnung der Strömungsverluste eines ebenen Schaufelgitters. Ing.-Archiv, Bd. XIX, Heft. 1, 1951, pp. 42-65. (On the Theoretical Calculation of Flow Losses of a Plane Cascade of Airfoils.)

Potential-flow solutions are used to determine the pressure and velocity distributions about the airfoils in two-dimensional cascades, from which the boundary layers on the blade surfaces can be determined by boundary-layer theory. Using zero-thickness profiles, the changes in exit-flow direction caused by the boundary layers can then be computed. Loss coefficients are found for eight different cascades, but no comparisons are made with experimental results.

79. Stephenson, J. M.: Efficiency and Drag of an Axial-Flow Compressor Stage. Aircraft Eng., vol. XXV, no. 292, June 1953, pp. 158-160.

The efficiency concepts frequently used in axial-flow-compressor design are reviewed. The relations are discussed between the various efficiencies defined, such as polytropic, isentropic, and cascade or blade efficiencies. The stage efficiency is expressed in terms of blade force and profile drag from considerations of the moment of momentum and the energy equations. The results can then be compounded for a complete stage. A merit figure for cascades in terms of a dimensionless force coefficient in the tangential direction and the profile drag coefficient is proposed.

An interesting discussion deals with the effects of errors in the profile shape produced on the efficiency. An overly thick blade trailing edge has little effect on the efficiency but causes the profile drag to increase. A blunted leading edge likewise has little effect on the design efficiency but does result in an abbreviated flow range before stall occurs. Sharp curvatures and ridges lead to a similar result. Surface roughness, if less than the boundary-layer thickness, has no effect. Reynolds number effects on the efficiency are reviewed briefly, and shed vortices and tip-clearance eddies and their effects are described.

80. Goldstein, Arthur W., and Mager, Artur: Attainable Circulation about Airfoils in Cascade. NACA Rep. 953, 1950. (Supersedes NACA TN 1941.)

A method is presented for determining the minimum required cascade solidity for a given turning. This method is developed for calculating the pitch-chord ratio and surface-velocity distribution consistent with specifications on profile thickness, maximum surface velocity, Reynolds number, inlet and outlet flow velocities and angles, suction-surface diffusion avoiding separation, and trailing-edge loading. The analysis is based on boundary-layer data obtained on isolated airfoils and approximate boundary-layer theory in two-dimensional incompressible flow. Both laminar and turbulent boundary layers are considered. Numerical examples are given, and results are compared with experiment.

81. Mager, Artur, Mahoney, John J., and Budinger, Ray E.: Discussion of Boundary-Layer Characteristics Near the Wall of an Axial-Flow Compressor. NACA Rep. 1085, 1952. (Supersedes NACA RM E51H07.)

The boundary-layer velocity profiles on the casing of an axial-flow compressor behind the guide vanes and rotor are measured by means of total-pressure and claw-type probes. It is assumed that the probes read the correct mean value of the flow in the presence of disturbances due to turbulence and blade rotation. The boundary-layer flow is resolved into two components, along the main flow streamlines and perpendicular to them.

A simplified analysis of the boundary-layer flow is undertaken, assuming incompressibility and zero velocity gradient normal to the surface at the boundary-layer outer extremity. Boundary-layer thickness and deflection at the wall are used as generalizing parameters. The experimental results and the momentum-integral equations are used to evaluate qualitatively the compressor wall boundary-layer characteristics.

The main parameters controlling secondary flow appear to be the turning of the flow and the product of the boundary-layer thickness and streamline curvature outside the boundary layer. Tip clearance affects secondary flow primarily at high weight flows and speeds. Losses near the rotor tip and stator hub are traced to the predominantly tangential direction of the boundary-layer flows in those locations.

Separation of the three-dimensional boundary layer is discussed. Preliminary considerations indicate that many phenomena in axial-flow compressors, such as large losses near the rotor tips despite energy addition to the boundary layer, can be explained by the three-dimensional boundary-layer flow on the casing.

82. Sinnette, John T., Jr., and Costello, George R.: Possible Application of Blade Boundary-Layer Control to Improvement of Design and Off-Design Performance of Axial-Flow Turbomachines. NACA TN 2371, 1951.

Assuming a linear pressure-volume relation for compressible flow, a potential-flow method is developed for designing blades slotted for suction or ejection, with a prescribed velocity distribution on the blade surface and in the slot, which is intended for use as boundary-layer control in the compressor or turbine. The effects of suction on the flow near the slot and of ejecting gas at temperatures and pressures different from the local free-stream values are discussed. Suction is recommended for boundary-layer control in later stages of compressors, and ejection is recommended for cooling purposes and for increasing the loading in turbine stages.

83. Scholz, N.: Two-Dimensional Correction of the Outlet Angle in Cascade Flow. Jour. Roy. Aero. Sci., vol. 20, no. 11, Nov. 1953, pp. 786-787.

An approximate method is developed for estimating a two-dimensional correction of the outlet flow angle of a cascade for the effects of boundary-layer growth on the end walls. It is assumed that the entire flow deflection takes place downstream of a plane within the passage, at the aerodynamic center of the passage. Momentum considerations then lead to correction in terms of flow contractions in two regions on either side of the plane.

84. Stephenson, J. M.: The Elimination of Wall Effects in Axial-Flow Compressor Stages. Jour. Aero. Soc. (London), vol. 57, no. 508, Apr. 1953, pp. 241-243.

A design method is proposed that improves the accuracy of the usual design assumptions for viscosity effects in compressors. The method relies upon the tendency of

gases to maintain a radial-equilibrium condition. This is used to counteract the peaking of the axial-velocity component due to wall friction (see survey 213). The boundary layers must be computed for the first stage, and then the results are applied to all stages with suitable corrections for the hub-tip radius ratios.

The procedure is as follows: A function  $f(r)$  is defined as a reaction parameter. For a given  $f(r)$ , the tangential velocities are calculated, and, assuming no radial-velocity components, the axial-velocity profile is determined. If it is desired to choose  $f(r)$ , that is, the tangential-velocity profile, a trial-and-error solution is set up in order to get a particular axial-velocity profile.

The over-all design method is intended to achieve two goals: (1) Each stage of a compressor must be designed with constant circulation to avoid the peaked axial-velocity profile generally found in compressors. A great deal of information is required about the profile in each stage which is assumed to be unchanged when different blading is used. (2) The axial velocity must be controlled without increasing the circulation near the wall, as this would cause an efficiency decrease due to secondary-flow effects.

85. Torda, T. P., Hilton, H. H., and Hall, F. C.: Analysis of Viscous Laminar Incompressible Flow Through Axial Flow Turbomachines with Infinitesimal Blade Spacing. Rep. No. C-TR-1, Eng. Exp. Station, Aero. Eng. Dept., Univ. Ill., Feb. 24, 1953. (Contract AF 33(616)-52, E.O. No. R-467-3 BR-1.)

An analysis is presented of the viscous steady laminar incompressible axisymmetric flow through rotating axial-flow turbomachines. The inverse or design problem is solved here. First, the resultant velocity distribution in each channel is assumed a nonsymmetric parabola with constant axial-velocity component, and the Navier-Stokes equations for axisymmetric flow are integrated. Introduction of a force field takes into account both hydrostatic pressures and viscous shear stresses, and makes possible the determination of the closely spaced three-dimensional streamline surfaces (i.e., the blades) by integrating the streamline equations along the stage. In this design problem, the angular velocity, the magnitude and direction of the entrance velocity, the entrance pressure, and the hub and shroud radii are given. From these, certain arbitrary functions  $Z_1$  and  $Z_2$  must be determined on the basis of known efficient surface curvatures at the leading edge. Such curvatures are chosen to conform with known physical limitations on the rate of pressure rise on blades necessary to prevent increased viscous losses and flow separation. The functions  $Z_1$  and  $Z_2$  also determine the twist of the blades. With this information, the designer can then determine the streamline traces by integration and thus the blade profiles for a given stage.

A numerical example is presented to illustrate use of the design procedure, its trends, and limitations. The particular numerical example in the report was one of a series whose over-all aim was to determine the conditions for maximum pressure rise through a single rotating stage of closely spaced blades at the same time to avoid large blade twists.

86. Torda, T. P., Hilton, H. H., and Hall, F. C.: Analysis of Viscous Laminar Incompressible Flow Through Axial-Flow Turbomachines with Infinitesimal Blade Spacing. Jour. Appl. Mech., vol. 20, no. 3, Sept. 1953, pp. 401-406.

Expressions are derived for the velocity components, pressure, and power input and output for arbitrary blade surfaces for viscous incompressible steady flows. The flow variables and blade surfaces are plotted.

87. Born, F. W., Hall, F. C., Hilton, H. H., and Torda, T. P.: Analysis of Viscous Laminar Incompressible Flow Through Axial Flow Turbomachines with Finite Blade Spacing. Tech. Rep. C-TR-2, Eng. Exp. Station, Aero. Eng. Dept., Univ. Illinois, Mar. 18, 1953. (Contract AF 33(616)-52, E.O. No. R-467-3 BR-1.)

The work of survey 85 is extended to the case of finite and equally spaced blades. In somewhat the same fashion as in surveys 10 to 12, a number of equally spaced streamline surfaces, calculated for infinitesimal spacing, are taken as "frozen," that is, the same for the finite as for infinitesimal spacing. The infinitesimal solution qualitatively determines the over-all properties of the stage. The flow variables are then allowed to vary from the frozen values circumferentially toward the surfaces of the adjacent finite-spacing blades by simultaneously replacing the Lorenz-type force field by series expansions for the axial-, radial-, and tangential-velocity components and the pressure. By substituting these series into the Navier-Stokes and continuity equations, recurrence formulas are obtained for the series coefficients. The first terms of the series are those obtained from axisymmetric solutions. The series for two of the three velocity-variable expansions have arbitrary functions for their second terms and must be determined from the boundary conditions. Once the coefficients are determined, the streamline equations are integrated, and the blade surfaces are found. The boundary conditions that must be satisfied are nonslip of flow along the blade surfaces and closure of the streamline surfaces to form finite blades.

Relaxation of the nonslip condition in prescribed fashion leads to a necessary simplification of both the analysis and the numerical work. The condition of closure is rigidly satisfied, but in general has to be fulfilled individually for each design in the course of the numerical work.

88. Torda, T. P.: Analysis of Viscous Laminar Incompressible Flow Through Axial Flow Turbomachines. Annual Summary Rep. C-TR-3, Eng. Exp. Station Aero. Eng. Dept., Univ. Illinois, July 15, 1953. (Contract AF 33(616)-52, E.O. No. R-467-3 BR-1.)

This third report in the series (surveys 85 and 87) is an exposition and summary of the first two reports.

#### G. BLADE ADJUSTMENT

89. Sinnette, John T., Jr., and Voss, William J.: Extension of Useful Operating Range of Axial-Flow Compressors by Use of Adjustable Stator Blades. NACA Rep. 915, 1948. (Supersedes NACA ACR E6E02.)

The inherently narrow operating flow and speed range of the axial-flow compressor is discussed. The useful flow range may be limited for one or more reasons, as follows: (1) a rapid change in pressure ratio for small changes in flow, (2) significant change in efficiency for change in flow, and (3) surging (i.e., unstable operation) when flow is decreased beyond certain limits. The useful speed range may be limited by rapid decreases in efficiency from peak values with change in speed. Chief responsibility for these limiting flow and speed characteristics is attributed to the reduction in the lift coefficients of some blade rows, occasioned by the occurrence of unfavorable angles of attack on those blades at the off-design operating conditions. Resetting the blade angles by blade adjustment as operating conditions are changed is suggested in order to maintain the lift coefficients as high as possible. Stator blade adjustment is recommended for reasons of mechanical simplicity in high-speed multistage compressors. A considerable section of the report is devoted

to presenting a clear physical picture of the flow mechanisms leading to the unacceptable unstable conditions at off-design operation. Particular attention is called to the importance of the large density changes in a multistage compressor, making it much more sensitive to flow changes than a single-stage unit.

A general theory is presented for resetting the compressor stator blade angles. For simplicity, the analysis is for one-dimensional compressible flow with an estimated polytropic exponent. Conditions calculated at the mean radius are assumed to represent the average conditions at each axial position. This restriction is necessary, because, while the blade angles can be adjusted to give a desired angle of attack for a given row of blades at some point along the blade span, the angles of attack at other radial positions cannot be chosen simultaneously unless the blade twist can be altered. Various flow and boundary-layer considerations lead to choosing the midspan point as the criterion for blade-angle adjustments required.

Calculation procedures are presented separately for the entrance guide vanes and the remaining stages, because the entrance guide vanes have functions essentially different from the typical stator blades. Then calculations are presented for resetting the blade angles of the NACA eight-stage compressor for 75 percent of design speed over a range of load coefficients from 0.28 to 0.70. The results show substantial improvement in the peak efficiencies at compressor speeds considerably below design speed. Peak pressure ratios were increased with the stator blades reset. Inlet-air conditions had a large effect on the adiabatic temperature-rise efficiency at low Mach numbers. The compressor had no definite surging below Mach number 0.4.

Stator blade adjustment can yield substantial improvement of peak efficiencies at speeds below the design speed. The availability of several different blade settings makes it possible to vary the peak-efficiency flows and thereby extend the useful compressor flow range.

90. Sinnette, John T., Jr.: Increasing the Range of Axial-Flow Compressors by Use of Adjustable Stator Blades. Jour. Aero. Sci., vol. 14, no. 5, May 1947, pp. 269-282.

A method is described to change blade settings in order to maintain optimum coefficients of lift and improved compressor performance for a range of speeds and flows (see survey 89). Tests on the NACA eight-stage compressor show that a substantial increase in useful operating range can be obtained in this fashion. The theory can be applied to rotor or stator blade adjustment, or both. The tests indicate that stator blade adjustment alone is sufficient to extend greatly the useful operating range of the compressor.

91. Jeffs, R. A., Hartley, E. L., and Rooker, P.: Tests on an Axial Compressor with Various Stator Blade Staggers. Memo. No. M.100, British N.G.T.E., Sept. 1950.

Low-speed tests were made on six stages of a medium-stagger free-vortex-design axial-flow compressor. The blading stagger was varied over a wide range, keeping the rotor blade angles fixed, in order to evaluate the effects of stagger changes in improving off-design operation.

As a result of the adjustments, efficiencies greater than 85 percent were obtained over a range of stator blade stagger angles from  $-50^{\circ}$  to  $10^{\circ}$ . Design stagger was  $-25.4^{\circ}$ . Estimated rotor and stator blade incidence angles indicate that the onset of surging can be regulated by controlling the stalling of stators or rotors. Stalling appears to be more critical on rotor than on stator blades.

92. McCoy, Allan W., and Hooper, Alton V.: Blade Adjustment in Axial-Flow Compressor Stages. Jour. Aero. Sci., vol. 20, no. 1, Jan. 1953, pp. 43-48.

A simplified analysis demonstrates the effectiveness of stator and rotor blade adjustment in providing volume-flow regulation in an axial-flow compressor. The effectiveness of blade control is first defined as the relative change of volume flow per unit of angle adjustment at constant pressure rise. Results show high values can be obtained by use of high flow and pressure coefficients. For any grid element, the effectiveness of blade control is a function of the reaction ratio and the flow and pressure coefficients. Low reaction ratio is used for stator blade control, and high for rotor blade adjustment.

A simplified analysis presents further information concerning the radial distribution of blade-control effectiveness (see survey 89). Stator blade adjustment achieves maximum control effectiveness at the blade root section for conventional type stages and more even distribution of effectiveness with rotor blade control. Special stage designs for uniform radial distribution of control effectiveness are discussed. To the reviewers, the assumptions in this last analysis, that the flow is incompressible, that the radial-flow components in the increased flow obtained by blade adjustment are similar to those obtained at design conditions for each blade element (i.e., no radial equilibrium), that the outlet-flow angles remain constant radially, and that the radial distribution of pressure rise remains constant, appear so large that they qualify the results.

93. McCoy, A. W., and Brunner, M. J.: The Use of Stator-Blade Control to Obtain Wide Range of Compressor Performance for Wind-Tunnel Application. A.S.M.E. Trans., vol. 76, no. 2, Feb. 1954, pp. 233-240.

Stator blade adjustment makes possible attainment of wide ranges of pressure ratio and volume flow at constant rotor speed, as compared with the use of stator blade adjustment to obtain favorable operation over a wide range of speed (see surveys 89 to 92). Design considerations and performance predictions for axial-flow compressors with stator adjustment are discussed, and experimental confirmation is provided.

#### H. ACTUATOR-DISK SOLUTIONS

94. Bragg, Stephen L., and Hawthorne, William R.: Some Exact Solutions of the Flow Through Annular Cascade Actuator Discs. Jour. Aero. Sci., vol. 17, no. 4, Apr. 1950, pp. 243-249.

The incompressible inviscid flow through a stage of an axial-flow turbomachine is approximated by the flow through an actuator disk, the limiting case when the chord of the blades approaches zero and the tangential velocity component changes discontinuously. As pointed out in survey 49, this assumption serves the mathematical purpose of making the flow equations (first made linear by assuming no self-transport of vorticity) homogeneous.

Two examples are chosen in which exact actuator-disk solutions and numerical solutions can be obtained. The results indicate that the radial flows decay rapidly with distance from the disk section. Nearly half the total radial-flow decay occurs within a distance of one tenth of the tip radius. For incompressible nonviscous flows, the value of the axial velocity at the disk section is almost exactly the mean between the values far upstream and downstream of the cascade.

The actuator-disk solution may be useful as a quick check on the more involved approximate methods such as in survey 49.

95. Siestrunk, R., and Fabri, J. (S. Reiss, trans.): Vortex Flow in Axial Machines. NACA Translation. (Trans. from Office Nat. D'Études et de Recherches Aero. Pub. No. 45, 1950.)

The linearized axisymmetric actuator-disk solution is extended to include compressible flows. The flow is characterized by a meridional stream function satisfying continuity, and a moment of momentum function with respect to the machine axis of rotation. A general relation between the stream and momentum functions is established for the two-dimensional compressible vortex flow. Linearized solutions are computed for incompressible flow, and the relations applied. Experimental deflection curves obtained from cascade tests are introduced with the solution procedure. Several nonpotential-flow cases are also calculated.

The differences between deflections obtained in flows in two-dimensional cascade tests and the three-dimensional flows in machines are discussed, as well as off-design point conditions in multistage machines.

96. Siestrunk, Raymond, et Fabri, Jean: Solution linearisée du mouvement tourbillonnaire d'un fluide compressible dans une roue axiale. Pub. Sci. et Tech., t. 1, no. 248, 1951, pp. 75-82. (Linearized Solution of the Turbulent Motion of a Compressible Fluid in an Axial-Flow Turbine.)

A lifting-line theory is developed for axisymmetric flow in an axial-flow turbine with cylindrical walls. Relations are obtained between the vorticity and enthalpy gradients. Stream functions are chosen to satisfy the relations. It is found that the linearized equations have simple solutions.

97. Raily, J. W.: The Flow of an Incompressible Fluid Through an Axial Turbomachine with any Number of Rows. Aero. Quart., vol. 3, pt. 2, Sept. 1951, pp. 133-144.

Following survey 49, a solution is proposed for nonviscous axisymmetric flows for cylindrical walls and their lifting disks. The continuity equation is integrated to obtain the axial velocity at the lifting section. This turns out to be the average of the upstream and downstream velocities. One solution for the equation of motion is obtained by neglecting radial displacements; another is obtained for the velocity distribution of a single blade row. Because the equations were linearized, the solutions for the flow fields to each row can be superposed to obtain the flow through a multistage unit.

Calculations by this method are affected by successive approximations. A computation is presented for a single compressor stage with untwisted blades and axial velocities upstream. Three approximations are computed and reasonable convergence is obtained.

# I. THREE-DIMENSIONAL FLOWS

98. Meyer, Richard: Beitrag zur Theorie feststehender Schaufelgitter. Nr. 11, Mitt. aus dem Inst. f. Aero (Zürich), 1946.

The first detailed analysis of the three-dimensional incompressible flow through a stator cascade with cylindrical walls is presented herein. The source and vortex method of survey 245 is employed to extend the solution from the infinite to the finite number of blades. The solution is exact for flows that are irrotational upstream and downstream of the rotor. They may be rotational within the rotor, but no vorticity is shed from the blades.

Briefly, the method modifies the solution for an infinite number of blades by a Fourier analysis to obtain the solution for a finite number of blades. Proper care is taken to prescribe the components of the force field or the energy distribution so that the force field will be perpendicular to the family of blade surfaces (Bauersfeld condition, 1905). This is the necessary condition for the infinite blade solution to be at all comparable with the nonviscous solution for finite number of blades. However, for large aspect ratios, the idealized flow will be comparable to the real flows even if this condition has not been fulfilled.

99. Dreyfus, L. A.: A Three-dimensional Theory of Turbine Flow and Its Application to the Design of Wheel Vanes for Francis and Propeller Turbines. Acta Polytech. Mech. Eng. Ser., vol. 1, no. 1, 1946.

The solution for an infinite number of blades is extended to a solution for a finite number of blades, and the results are discussed. A power-series development is employed, and its first term is determined from the solutions for infinite number of blades. The second term of the series is explicitly determined from the equations of continuity and irrotational absolute flow.

This report provided many of the ideas developed in surveys 10 to 12, 34, and 103.

100. Weske, John R.: Fluid Dynamic Aspects of Axial-Flow Compressors and Turbines. Jour. Aero. Sci., vol. 14, no. 11, Nov. 1947, pp. 651-656.

The stage velocity diagrams for the ideal operation of radial elements of an axial turbomachine stage (for flows confined to coaxial stream surfaces in rotationally symmetric flows with identical inlet and outlet velocities) can be completely defined in terms of three dimensionless parameters - flow, pressure, and reaction coefficients. The flow coefficient is the ratio of the axial velocity to the wheel speed. The pressure coefficient is the ratio of stage total-pressure change to wheel-speed velocity energy. The reaction coefficient is the ratio of rotor static-pressure change to stage total-pressure change. Radial variations of the pressure and reaction coefficients are found for two specific cases.

The three-dimensional aspects of the flow resulting from viscosity are qualitatively discussed. For example, radial variations of the circumferential velocity component, which can result from boundary-layer effects, cause corresponding changes in these fundamental parameters governing stage design. Considering the flow through the blade rows as flow in curved channels, the behavior and effects of the surface boundary layers, the blade wakes, and centrifugal action in the rotors are analyzed. A criterion for radial stability of the compressible flow on coaxial stream surfaces is proposed, which indicates that the stability depends on the radial distribution of total energy (as for incompressible flow) and also on the temperature distribution. With this criterion, the radial displacement of the boundary layer in turbomachines is analyzed for various conditions of flow circulation and boundary-layer distribution.

101. Kahane, A.: Investigation of Axial-Flow Fan and Compressor Rotors Designed for Three-Dimensional Flow. NACA TN 1652, 1948.

An analysis of the three-dimensional aspects of the flow in axial-flow turbomachines is directed to the problem of attaining higher stage pressure ratios. An approximate method is developed to adapt the two-dimensional airfoil data for use in three-dimensional flow considerations. The results are as follows: (1) Higher pressure ratios than possible with free-vortex design can be obtained by loading the



rotors at the tip sections. The flow, however becomes three-dimensional. (2) High-efficiency axial-flow compressors can be achieved with three-dimensional flows. (3) The three-dimensional theory based on two-dimensional-cascade data is sufficiently accurate for design purposes. (4) Tip-clearance losses of highly loaded rotor blades are not excessive.

102. Schnittger, J. R.: Three-Dimensional Flow in Axial Compressors. Eng. Digest, vol. 13, no. 3, Mar. 1952, pp. 89-90.

The three-dimensional flow characteristics in axial-flow compressors with axisymmetric flows are determined by applications of the vortex-field and radial-equilibrium theories. Under vortex-field theories, a brief discussion is presented of (1) calculations of potential, incompressible flows with constant axial-velocity components and constant whirl radially (by the method of survey 23), and (2) vortex flow showing velocity distributions (calculated by the method of survey 49).

Calculations based on radial-equilibrium conditions indicate methods to obtain expressions for radial acceleration, energy gradients, and interference between cascades. Methods based on the radial-equilibrium equation are easy to handle and can account for compressible-flow and frictional effects.

103. Wu, Chung-Hua: A General Theory of Three-Dimensional Flow in Subsonic and Supersonic Turbomachines of Axial-, Radial-, and Mixed-Flow Types. NACA TN 2604, 1952.

A general three-dimensional nonviscous compressible-flow theory is developed for subsonic and supersonic turbomachines with finite numbers of blades of finite thickness and arbitrary hub and casing shapes. The combined theory is applicable to axial-, radial-, or mixed-flow turbomachines for both the direct and inverse problems. Such a theory is required for any kind of an accurate representation for cases of low hub-tip radius ratios, for cases of high inlet Mach number, and highly loaded stages where two-dimensional solutions are inadequate.

The three-dimensional solution is obtained in an essentially two-dimensional manner. Solutions for mathematically two-dimensional flows on two different kinds of relative stream surface are combined by an iteration process. A relative stream surface of the first kind  $S_1$  extends from the suction surface of one blade to the pressure surface of the adjacent blade. The blade-to-blade flow variations can be computed on such surfaces. Twist of the  $S_1$  surfaces may lead to large values of circumferential derivatives. A relative stream surface of the second kind  $S_2$  is one between two blades, extending from hub to outer casing. The through-flow solution (survey 34) is a special case of an  $S_2$  stream-surface solution. The equation of continuity is combined with the appropriate equation of motion in either the tangential or radial direction through use of a special stream function defined on the surface. A nonlinear partial-differential principal equation of flow results. The equations obtained to describe the flow on these mean stream surfaces show clearly the approximations involved in ordinary two-dimensional solutions. The character of the nonlinear partial-differential equation, whether elliptic or hyperbolic, depends upon the relative magnitude of the local velocity of sound and certain combinations of velocity components of the fluid.

General methods of solution of the equations by hand-operated or by high-speed digital computing machines are presented. In general, the three-dimensional solutions for both the direct and the inverse problems employ the solutions on both kinds of relative stream surfaces. The correct solution on one kind of surface often requires information obtained from solutions on the other kind. Consequently, an

iterative solution process between the two kinds of surfaces may be required. In the direct problem, the solution starts with an assumed flow surface and proceeds with alternate solutions on the two kinds of flow surfaces until a satisfactory approximation is obtained. The better the first approximation (see survey 17), the shorter the computation.

The process is shorter for the inverse problem. The calculation begins on the mean  $S_2$  surface. The designer can specify one degree of freedom and an estimated blade-thickness distribution. After the solution on this  $S_2$  surface has been obtained, the blade coordinates are determined by extending the solution circumferentially on an  $S_1$  surface.

The analysis of the three-dimensional theory can provide a clearer understanding of the flows in a turbomachine than was obtainable by more simplified solutions, in the opinion of the reviewers. More complete knowledge about the behavior of the main stream and its effects on the development of viscous boundary layers might then aid in understanding the secondary-flow behavior in turbomachines. This theory is applicable to both irrotational and rotational absolute flow at the inlet and at both design and off-design conditions. However, because of the formidability of the complete three-dimensional design theory involving iterative solutions between the  $S_1$  and  $S_2$  surfaces, its application to compressor design is impractical at this time. Little has been recorded about its actual use. At present, then, the three-dimensional theory can best serve as a useful guide for evaluation of experimental data.

104. Wu, Chung-Hua: Matrix and Relaxation Solutions that Determine Subsonic Through Flow in an Axial-Flow Gas Turbine. NACA TN 2750, 1952.

The usefulness of the three-dimensional flow theory (survey 103) depends greatly on the ease of obtaining solutions on the  $S_1$  or  $S_2$  relative stream surfaces. The principal flow equations for both kinds of surfaces and the methods of successive approximations used for their solution are similar. Their ease of solution and rate of convergence can be expected to be nearly equal.

This report presents and evaluates three methods for obtaining solutions on  $S_2$  surfaces, namely, (1) relaxation method for hand-operated desk calculator, (2) matrix method on an IBM card-programmed electronic calculator, and (3) matrix method on Univac.

The incompressible and compressible nonviscous flows are computed for a single-stage axial-flow turbine of free-vortex velocity distribution on the  $S_2$  surface with cylindrical bounding walls and a hub-tip radius ratio of 0.6. Convergence is readily obtained. The matrix methods proved quicker and more accurate, and calculations are not overly difficult, according to the author. The results of these calculations can be used to evaluate simpler, more approximate methods for computing subsonic through-flows in turbomachines.

Interesting flow properties are also brought to light by the calculations. For example, the actual flow path through a blade row, which may be far from sinusoidal (see surveys 33, 36, and 6), depends upon such factors as the shape (radial twist) of the stream surface and the compressibility of the gas. The shape of the stream surface is particularly sensitive to the axial position of the radial element of the stream surface. Upstream of the radial element, radially inward flows occur; downstream, radially outward flows occur. For incompressible flows, considerable radial flow results because of the circumferential pressure gradient and the radial twist of the stream surface. The flow is radially inward in the stator and outward in the

rotor of this turbine. The effects on the flow distribution are significant. Large negative radial gradients of axial velocity result in the space between the stator and rotor. For compressible flows, the effects of compressibility and the radial twist on the flow distribution are nearly equal. The nonlinear effects of the governing equations are particularly evident here.

Assuming a sinusoidal flow path within a blade row can apparently provide a reasonable measure of the radial displacement effects for calculation purposes (survey 36). However, the actual flow path may not be sinusoidal.

105. Marble, Frank E., and Michelson, Irving: Analytical Investigation of Some Three Dimensional Flow Problems in Turbomachines. NACA TN 2614, 1951.

This report extends the linear theory to the study of off-design operation and mutual interference. The flow is considered axisymmetric incompressible and non-viscous in an axial-flow turbomachine. The radial-velocity components and the deviation of the axial velocity from the mean through-flow are assumed small. The theory points to three additive flow components: (1) uniform through-flow, (2) radial-equilibrium solution, correct upstream and downstream, and (3) a "fine-structure" accounting for accelerations. Tables are given for calculating the fine-structure by punch-card methods for a hub-tip radius ratio 0.6.

An actuator-disk approximation is made to the fine-structure with numerical examples. The discussion from this includes (1) transients in the first few stages of a multistage unit, (2) fluctuations of axial-velocity distribution within a machine, (3) the performance of a single blade row with a prescribed distribution of trailing-edge flow angle and dependence upon radius and aspect ratio, and (4) off-design performance of blade rows and mutual interference effects.

The theory involves a second-order linearization in order to handle problems concerning flows with greater vorticity effects than could be assumed in neglecting self-transport of vorticity (see survey 49).

106. Stanitz, John D., and Ellis, Gaylord O.: Flow Surfaces in Rotating Axial-Flow Passages. NACA TN 2834, 1952.

The significance of the effects of radial twist of the relative stream surfaces on the radial distribution of velocities downstream of a blade row are discussed in survey 104. The present investigation attempts to discover the magnitude of the actual deviations of the flow surfaces from their assumed orientation in typical two-dimensional solutions. The flow is assumed incompressible, nonviscous, and absolute irrotational through rotating finite-spaced straight blades of infinite axial length. The blades have no spanwise loading variations, which linearizes the solutions. Numerical solutions are obtained for five passages for a range of blade spacing and hub-tip radius ratio. The solutions are found by superposing the solution for zero through-flow in passages when rotating upon solutions for through-flow in the passages where stationary. In this fashion all ratios of axial velocity to passage tip speed can be accommodated.

107. Yamancuchi, Masao: Three Dimensional Considerations of Flow Through Turbines. Rep. No. 3, Trans. Tech. Res. Inst., Oct. 1952. (Pub. by The Unyu-Gijutsu Kenkyūjo Mejiro, Toshimaku, Tokyo, Japan.)

Flow with axial symmetry, small secondary-flow effects, viscous losses in the stream recovered as temperature rise, and slight streamline deviation through the passage is assumed. In the design and performance analysis of turbines, the performance of the stage is usually considered to be fairly represented by that of the blade

elements at the mean pitch circle radius. This analysis intends to evaluate the closeness of such approximations and to discuss their satisfactoriness.

The three-dimensional flow in a turbine due to variations of velocity coefficients of the blade elements and of the circulation spanwise is discussed. Fundamental equations describing the flow patterns are derived. Three numerical examples of constant circulation radially, constant nozzle discharge angles, and uniform axial turbine discharge are provided. The usual simplified design procedures are too inaccurate to permit making an advanced, high-performance turbine.

108. Holmquist, Carl O.: An Approximate Method of Calculating Three-Dimensional, Compressible Flow in Axial Turbomachines. Ph.D. Thesis, C.I.T., 1953.

An approximate solution is presented to the direct and inverse compressible-flow problem in arbitrarily shaped axisymmetric channels. The flow is steady, axisymmetric, compressible, and nonviscous. Real-flow effects are approximated by use of a polytropic exponent. The blades are radial. No boundary-layer effects or blade interference are considered. By consideration of the upstream conditions, the channel configurations, and the blade shapes, the flow immediately downstream of the blade row is analyzed. An integral for the axial velocity is derived from the known upstream conditions and the downstream static pressure. The total-pressure distribution is determined by integrating the radial-momentum equation and by applying the energy equation at the boundary. The mass-flow equation is solved for the value of the constant obtained by integrating the axial velocities with an assumed distribution. An iteration process is set up in which the axial-velocity distribution is assumed, and the procedure leads to a new axial-velocity distribution. The process continues until a velocity distribution produced by an iteration equals the assumed distribution.

## SECTION II. EXPERIMENTAL PERFORMANCE AND CHARACTERISTICS

### A. AIRFOILS AND BLADE SECTIONS

109. Abbott, Ira H., von Doenhoff, Albert E., and Stivers, Louis S., Jr.: Summary of Airfoil Data. NACA Rep. 824, 1945. (Supersedes NACA WR L-560.)

Airfoil data for flight and wind-tunnel tests obtained in the NACA Langley two-dimensional low-turbulence pressure tunnel are collected and correlated. Flight-data drag measurements were obtained by wake-survey methods. Included are analyses of the lift, drag, pitching moment, critical-speed characteristics, pressure and velocity distributions, and a discussion of the effects of surface conditions.

110. Loftin, Laurence K., Jr.: Theoretical and Experimental Data for a Number of NACA 6A-Series Airfoil Sections. NACA Rep. 903, 1948. (Supersedes NACA TN 1368.)

Experimental results obtained in a two-dimensional investigation of five NACA 64A-series and two NACA 63A-series airfoil sections are presented. The NACA 6A-series airfoils were designed to eliminate the trailing-edge cusp of the NACA 6-series airfoils by making the sides straight from 80 percent of chord to the trailing edge. The data were obtained for the NACA 6A-series basic thickness forms with the minimum-pressure point at 30, 40, and 50 percent of chord, for thickness ratios from 6 to 15 percent of chord, and at Reynolds numbers of  $3 \times 10^6$ ,  $6 \times 10^6$ , and  $9 \times 10^6$ .

The test results indicate that the section minimum drag and maximum lift of comparable NACA 6-series and 6A-series airfoils are nearly the same. The lift-curve

slope of the NACA 6A-series is smooth and nearly independent of the airfoil thickness ratio, in contrast to trends of the NACA 6-series airfoils. Leading-edge roughness causes the lift-curve slope of the 6A-series sections to decrease as the airfoil thickness ratio increases.

111. Beaven J. A., Sargent, R., North, R. S., and Burrows, P. M.: Measurements of Maximum Lift on 26 Aerofoil Sections at High Mach Number. R. & M. No. 2678, British A.R.C., 1948.

Tests were conducted on 2-inch-chord airfoil sections at Reynolds numbers up to 750,000, Mach numbers up to 0.85, and angles of attack up to  $15^\circ$ . Conventional, NACA, and low-drag airfoils were tested. The test results are presented as curves of lift coefficient against incidence angle for a range of Mach numbers, and curves of the maximum lift coefficient against the Mach number for each airfoil.

In general, the curve of lift coefficient against Mach number shows a peak characteristic at low Mach numbers. At higher Mach numbers of approximately 0.75, the curve shape changes to a continuous rise of lift coefficient with angle of attack. Below this critical Mach number of 0.75, the maximum lift coefficient of each airfoil decreases as the Mach number increases. At Mach numbers greater than about 0.6, the section characteristics conducive to increasing the maximum lift are thinness, higher camber, and rearward location of the maximum camber point (see surveys 125, 126, and 118).

112. Loftin, Laurence K., Jr., and Smith, Hamilton A.: Aerodynamic Characteristics of 15 NACA Airfoil Sections at Seven Reynolds Numbers from  $0.7 \times 10^6$  to  $9.0 \times 10^6$ . NACA TN 1945, 1945.

The two-dimensional aerodynamic characteristics of ten NACA 6-series sections and five NACA 4-digit and 5-digit series airfoils are presented. The data show the effects of systematic variation in airfoil thickness, thickness distribution, and camber at each of seven Reynolds numbers covering the range from  $0.7 \times 10^6$  to  $9.0 \times 10^6$ . The maximum Mach number was 0.15, and correction factors were applied for the boundary layer. The tests were conducted on both smooth and rough surfaces.

The results show that the drag coefficient at design coefficient of lift conditions and the maximum coefficient of lift are the most important aerodynamic characteristics affected by Reynolds number variations. For all the airfoils tested, the drag coefficient at design lift increases as the Reynolds number decreases at all conditions. For smooth NACA 6-series airfoils, the drag-coefficient increase becomes larger as the airfoil thickness increases, and as the position of minimum pressure moves rearward on the basic thickness form at zero lift. For both rough surfaces and lower Reynolds numbers with smooth surfaces, there is no reduction in the minimum drag by use of the 6-series in lieu of the 5-series airfoils.

As the Reynolds numbers were decreased, the maximum lift for all the airfoils decreased in unpredictable fashion. In general, the extent of the low-drag range increased for the smooth 6-series airfoils as the Reynolds number decreased. For all airfoils, the extent of the low-drag range was greater than predicted theoretical values at the lower Reynolds numbers.

Some decrease of lift-curve slopes was observed as the Reynolds numbers decreased, but the angle of zero lift appeared almost independent of Reynolds number.

113. Schaefer, Raymond F., and Smith, Hamilton A.: Aerodynamic Characteristics of the NACA 8-H-12 Airfoil Section at Six Reynolds Numbers from  $1.8 \times 10^6$  to  $11.0 \times 10^6$ . NACA TN 1998, 1949.

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CP-6, back.

Similar to the material presented by survey 112, the NACA 8-H-12 airfoil was tested both smooth and roughened. Section lift, drag, and pitching-moment coefficients were obtained for a range of six Reynolds numbers from  $1.8 \times 10^6$  to  $11.0 \times 10^6$ , and the results are compared with data on NACA 0012 and NACA 23012 airfoil sections, both commonly used in rotor blades.

No unusual scale effects were observed for the smooth 8-H-12 blades over the Reynolds number range. Except for an adverse scale effect on the drag in the range of Reynolds numbers between  $2.6 \times 10^6$  and  $3.0 \times 10^6$ , the same is true for the roughened-edge airfoils. This effect is attributed to differences in the extent of roughness employed at those Reynolds numbers.

114. Smith, Hamilton A., and Schaefer, Raymond F.: Aerodynamic Characteristics at Reynolds Numbers of  $3.0 \times 10^6$  and  $6.0 \times 10^6$  of Three Airfoil Sections Formed by Cutting Off Various Amounts from the Rear Portion of the NACA 0012 Airfoil Section. NACA TN 2074, 1950.

Airfoil sections were formed by cutting off 1.5-, 4.0-, and 12.5-percent chord from the trailing edge of the NACA 0012 airfoil section. The 1.5- and 4.0-percent cases were tested at Reynolds numbers of  $3.0 \times 10^6$  and  $6.0 \times 10^6$ , and the 12.5-percent case was tested at a Reynolds number of  $6.0 \times 10^6$ . The 1.5-percent case was then roughened by having rivet heads attached near the trailing edge and tested, and tunnel wall and boundary-layer corrections were applied. The tests were conducted at low Mach numbers.

As the trailing-edge thickness increased, the maximum section lift coefficient varied little for the smooth condition but increased for the roughened condition, and the section drag coefficient increased uniformly over a wide range of lift coefficients.

The value of quarter-chord pitching moment at zero angle of attack remained approximately zero with increase of trailing-edge thickness, but the aerodynamic center moved rearward.

115. Amick, James L.: Comparison of the Experimental Pressure Distribution on an NACA 0012 Profile at High Speeds with That Calculated by the Relaxation Method. NACA TN 2174, 1950.

Pressure-distribution measurements were made on an NACA 0012 airfoil (5-in. chord) at zero angle of attack for a range of Mach numbers up to 0.75. The data are compared with calculations made by a relaxation method. At the lower Mach numbers, good agreement was achieved between the experimental and theoretical values, the spread in values increasing with Mach number. At the highest Mach number, the agreement was poor, perhaps as a result of the probable occurrence of weak shocks. At Mach number 0.7, the pressure distribution, calculated by applying the von Kármán-Tsien compressibility correction to the relaxation solution for the airfoil at zero Mach number in free air and modifying for tunnel constriction, approximates fairly well both the experimental pressure distributions and the relaxation-method pressure distributions for the airfoil in free air modified for tunnel constriction.

116. McCullough, George B., and Haire, William M.: Low-Speed Characteristics of Four-Cambered, 10-Percent-Thick NACA Airfoil Sections. NACA TN 2177, 1950.

The stalling characteristics of cambered airfoil sections, as affected by NACA 0010 and 64A010 basic thickness distributions, were investigated at low speeds and at Mach numbers of 0.131 and 0.187. The lift, drag, pitching-moment characteristics, and chordwise distribution of pressure were obtained for 10-percent-thick sections of these thickness distributions, each cambered with the same type mean line for design lift coefficients of both 0.3 and 0.8.

The results indicated that stall was affected little by the changes of thickness distribution but was affected by the camber. Visual indications by means of tufts showed that the stall of sections cambered for ideal lift coefficient (0.3) resulted from separation from the leading edge, which occurred almost immediately after the appearance of turbulent separation at the trailing edge. With sections cambered for ideal lift-coefficient (0.8), the turbulent separation from the trailing edge progressed almost as far forward as the 70-percent-chord station before laminar separation occurred at the leading edge. The NACA 0010-series sections had higher maximum lift than the NACA 64A010-series sections, but the difference was smaller between the more highly cambered sections. The maximum lifts of both series were reduced by surface roughness. The effects of Reynolds number variations on maximum lift were small for the range investigated.

117. Douglas, Ola: A Series of Low-Drag Aerofoils Embodying a New Camber-line. R. & M. No. 2494, British A.R.C., 1947.

A method developed in 1947 is applied to the design of camber lines. Better than the constant-load type, the new camber line includes larger values of lift-coefficient range and leading-edge curvature. Numerical methods are described which use Hollerith punch-card machines. A series of airfoils was designed and their characteristics displayed.

118. Van Dyke, Milton D.: High-Speed Subsonic Characteristics of 16 NACA 6-Series Airfoil Section. NACA TN 2670, 1952.

The high-subsonic speed characteristics were measured for NACA 63-, 64-, 65-, and 66-series airfoil sections with thickness ratios of 6, 8, 10, and 12 percent and ideal lift coefficient of 0.2.

Only slight impairment of the high-speed section drag characteristics results from movement up to 40 percent of chord forward of the position of minimum base-profile pressure. The decrease of lift-curve slope and the increase of angle of zero lift are delayed further beyond the critical Mach number. Therefore, for the 6-series airfoil sections with given thickness ratio, the optimum sections are obtained with the minimum-pressure point near 40-percent chord (see surveys 125 and 126).

The high-speed drag characteristics of the airfoil sections could be improved only by decreasing the thickness ratio, which led to a reduction in the range of coefficient of lift less severe than had been predicted. Even the thinnest sections maintained good high-speed performance over a wide range of lift coefficient.

#### B. STATIONARY BLADE ROWS

119. Kantrowitz, Arthur, and Daum, Fred L.: Preliminary Experimental Investigation of Airfoils in Cascade. NACA WR L-231, 1942. (Supersedes NACA CB.)

An experimental study was made at very low flow speeds through a stationary cascade of 65 2-810 blower-blade sections. The solidity was 1.0, the stagger  $45^\circ$ . The turning-effectiveness, pressure-distribution, pressure-rise, and lift and energy-loss characteristics were evaluated.

The turning angle for cascades of small-camber blades with solidity near 1 is approximately the blade angle of attack less the angle of attack for zero lift of the isolated airfoil. A large part of cascade losses is associated with flows along the channel walls and particularly with a region of slow air near the junctures of the blade convex sides with the walls (see surveys 230 to 234).

120. Zimney, Charles M., and Lappi, Viola M.: Data for Design of Entrance Vanes from Two-Dimensional Tests of Airfoils in Cascade. NACA WR L-188, 1945.

A series of blower-blade sections was developed for turning air efficiently from  $0^\circ$  to  $80^\circ$ . Tests were made of five NACA 65-series blower blades and of four experimentally designed blower blades. The turning effectiveness and pressure distribution of the blades were evaluated by stationary cascade tests of the blades at solidities nearly equal to 1 and at low Mach numbers.

Entrance-vane design charts, based on the two-dimensional-cascade tests, are presented for designing a blade section at a specified angle of attack with any desired turning angle. These blades operate with peak-free pressure distributions. The critical Mach numbers can be calculated approximately from the pressure distributions.

121. Bogdonoff, S. M., and Bogdonoff, H. E.: Blade Design Data for Axial-Flow Fans and Compressor. NACA WR L-635, 1945.

An experimental investigation was conducted to obtain blade design data suitable for high-efficiency axial-flow fans and compressors. The tests were conducted in a low-speed two-dimensional-cascade tunnel at Mach numbers of about 0.1 and at Reynolds numbers of about 300,000. Boundary-layer suction slots were used. Effects of camber, solidity, and stagger on the blade turning angle and shape of pressure-distribution curves were studied for a family of five low-drag airfoils. The airfoil cambers were varied to obtain a range for a free-air lift coefficient from zero to 1.8. Tests were then made at stagger angles of  $45^\circ$  and  $60^\circ$  and at solidities of 1.0 and 1.5. From these tests, blade design charts were prepared to give the camber and angle of attack setting for any desired turning angle. Blades thus chosen have nearly flat pressure-distribution curves. Some blades tested in a single-stage blower reached their maximum efficiency for operation near the flat pressure-distribution range. Empirical equations are presented by which the performance of airfoils in similar cascades can be predicted sufficiently accurately for blade design purposes.

122. Katzoff, S., Bogdonoff, Harriet E., and Boyet, Howard: Comparisons of Theoretical and Experimental Lift and Pressure Distributions on Airfoils in Cascade. NACA TN 1376, 1947.

One cascade of turbine blades, two cascades of entrance vanes, and three cascades of blower blades using NACA 6-series airfoil sections were tested. Most of the experimental data were taken from surveys 121 and 120. The experimentally observed lift coefficient was smaller than that calculated theoretically. These differences were greater in cascades than the corresponding differences in theoretical and experimental lift coefficients for isolated airfoils. The experimentally and theoretically determined pressure distributions also differed. However, when the theoretical lift was made to equal the experimental lift (by ignoring the Kutta condition or other methods), the pressure distributions agreed fairly well. The difficulties may have resulted from end effects, and the pressure distribution on a blade section in an actual blower may be considerably different from that in a cascade.

123. Bogdonoff, Seymour M.: NACA Cascade Data for the Blade Design of High-Performance Axial-Flow Compressors. Jour. Aero. Sci., vol. 15, no. 2, Feb. 1948, pp. 89-96.

The important blade design parameters (i.e., turning angles or loading), design angles of attack, and critical speed were evaluated. Tests made in two-dimensional cascades at low and high airspeeds show that the data obtained in the low-speed tunnel were suitable for high-speed conditions and for blades mounted in a rotor. It is estimated that the number of compressor-stages currently required could be halved by using highly loaded blades.



124. Katzoff, S., and Hannah, Margery E.: Further Comparisons of Theoretical and Experimental Lift and Pressure Distributions on Airfoils in Cascade at Low-Subsonic Speed. NACA TN 2391, 1951.

In this report, which supplements survey 207 and extends the results of survey 122, the lift and velocity distribution are compared for five highly cambered compressor-type blades of the NACA 6-series. Improved experimental data-taking techniques (survey 207) resulted in obtaining closer agreement between theoretical and experimental values. At Reynolds numbers of 250,000 and inlet velocities of about 95 feet per second, the experimental lift coefficients were less than the theoretical. The differences were larger for the more highly cambered blades and high-pressure-rise conditions. The pressure distribution calculated by equating the circulation to the experimental value and neglecting the Kutta condition agreed with the experimental distribution, provided the boundary layer did not separate.

125. Carter, A. D. S.: Some Tests on Compressor Cascades of Related Aerofoils Having Different Positions of Maximum Camber. Rep. No. R.47, British N.G.T.E., Dec. 1948.

Experimental investigations show that blades having their position of maximum camber well forward have a wide operating range and high choking mass flow but a low critical Mach number, based on drag rise. Shifting the position of maximum camber rearward narrows the working range but at the same time increases the critical Mach number, thus enabling increased work output for the corresponding compressor stage.

The position of maximum camber at 50-percent chord is a fair compromise (see survey 118). In actual compressor stages where secondary-flow effects may be large, these results may not apply.

126. Carter, A. D. S., and Hughes, Hazel P.: A Theoretical Investigation into the Effect of Profile Shape on the Performance of Aerofoils in Cascade. R. & M. No. 2384, 1950, British A.R.C.

A limited theoretical investigation was conducted into the effects of profile shape on the performance of airfoils in two-dimensional cascades. Parabolic- and circular-arc cambers were compared. The optimum maximum-thickness position and the desirable maximum thickness were likewise investigated. At low flow speeds, the parabolic-arc camber had a larger working range and was superior to the circular-arc camber airfoil. At high speeds, the circular-arc camber is better, because of the low critical Mach number of a parabolic-arc camber airfoil.

Moving the maximum-thickness position rearward helps by raising the critical Mach number but hinders by making the low-speed performance worse. The most suitable compromise for the best combination of good working range and fairly high critical Mach number appears to be with the position of maximum thickness at 40 percent of chord (see surveys 125, 111, and 118). Maximum thicknesses greater than 10 percent were found undesirable for cascade use.

127. Marcinowski, H.: Messungen an Axialen Schaufelgittern für Verzögerte Strömung. J. M. Voith (Heidenheim), VL-13-105, Mar. 1946. (Cascades of Airfoils in Axial Compressors.)

An investigation was made to determine the best blading for the last stator rows of multistage axial compressors. The effects of camber ratio, Reynolds number, and Mach number variations are plotted as curves of lift coefficient against angle of attack. Results indicate the desirability of using thinner blades at lower solidities.

128. Shimoyama, Yoshinori: Tests of Cascades of Airfoils for Retarded Flow. NACA TM 1190, 1947.

Wind-tunnel tests were conducted on airfoil cascades in decelerating flows corresponding to conditions in an axial-flow compressor rotor. Single-airfoil results in terms of lift and pressure-drag coefficients compare favorably with results from tests on mean sections of the airfoils in cascade.

129. Weinig, F., and Eckert, B.: Measurements on Compressor-Blade Lattices. NACA TM 1193, 1948.

Results of tests on the effects of Mach number on compressor blade lift and drag are presented. The flow directions were indicated visually upstream and downstream of the cascade by means of streamers. Because short blades were used, the influence of the boundary layer on both the inner and outer walls of the circular cascade tunnel was considerable. Plots are presented of the curves of lift coefficient against angle of attack and drag coefficient.

130. Davis, Hunt: A Method of Correlating Axial-Flow-Compressor Cascade Data. A.S.M.E. Trans., vol. 70, no. 8, Nov. 1948, pp. 951-955.

An empirical method is developed for correlating wind-tunnel tests on two-dimensional airfoil cascades. Two correlation charts relating camber, solidity, and stagger angle with entrance angle and turning angle are presented. Given any four of these variables, the fifth can be found by means of the charts. The construction of the charts is explained, and the sample set (for NACA 4-digit series) provided was based on tests of 40 different cascades at low Mach numbers. The sample set can be used directly for Mach numbers less than 0.4. For subsonic Mach numbers greater than 0.4, a simple computation scheme based on Eckert's adaptation of the Prandtl correction is provided to correct the solution. For each chart all the profiles belong to one family of airfoils; a new chart would be required for each new family.

Because the profile drag is not explicitly involved, it may require several trials of camber, solidity, and stagger variations to get an airfoil with satisfactory drag characteristics.

131. Ackeret, J., and Rott, N.: Über die Strömung von Gasen durch ungestaffelte Profilgitter. Schweiz. Bauzeitung, Jahrg. 67, 1949, pp. 40-41; 58-61. (Concerning the Flow at Gases Through Non-Staggered Grids.)

Symmetrical airfoils were investigated at five angle settings for a range of thickness ratios. The measurements included total and static pressures behind the trailing edges and static pressures on the airfoil surfaces and at the plane of symmetry between the airfoils. Schlieren patterns are presented of the shock formations, and formulas for choking Mach numbers and maximum drag are derived. Theoretical and experimental results agreed well.

132. Sawyer, William T.: Experimental Investigation of a Stationary Cascade of Aerodynamic Profiles. Mitt. aus d. Inst. f. Aero., Heft. 17, Zurich, 1949.

The effects of Reynolds number and turbulence on typical turbine blading were studied. A two- and three-dimensional experimental evaluation of the continuum-cascade method (survey 245) of blade design is made. All tests were conducted at very low Mach numbers, 0.1 or less, for small pressure changes across the cascade. Hence, the flow is considered incompressible. An experimental correlation is made of the static pressures at blade midspan and of the blade forces.

The results and conclusions are as follows: (1) Direct measurements of cascade blade forces by use of a suitably accurate balance was found feasible; (2) a valid practical blade efficiency in terms of force components was defined for the cascade, which was used to reflect the influence of the flow variables investigated; (3) the minimum Reynolds number for satisfactory cascade performance was identified with lift breakdown; (4) various means of observing boundary-layer transition and separation gave consistent results; and (5) the two-dimensional flow theory (survey 245) was confirmed, a cascade produced according to this theory having a mean turning angle within 1.1 percent of the desired turning angle. Discussions are presented about blade wakes, loss, turning angles, flow along blade surfaces, and tip-clearance effects.

133. Finger, Harold B., Schum, Harold J., and Buckner, Howard A., Jr.: Experimental and Theoretical Distribution of Flow Produced by Inlet Guide Vanes of an Axial-Flow Compressor. NACA TN 1954, 1949.

The performance of the inlet guide vanes of a multistage axial-flow compressor was investigated experimentally and compared with the performance predicted on the basis of a cylindrical-cascade evaluation method. Experimentally, the outlet flow angle was found to be independent of the weight flow. The deviation angle increased near the hub and tip but remained constant over the main portion of the blades. For camber angles greater than  $30^\circ$  in this configuration, a critical Mach number was attained at a weight flow of 38.9 pounds per second per square foot, at which the lift coefficient begins to decrease with increasing Mach number. The magnitudes of the flow velocities out of a blade row are calculated theoretically when given the flow angle and the total-enthalpy distributions. The method involves relations obtained from the energy, continuity, and simplified-radial-equilibrium conditions.

134. Constant, R.: Note on Performance of Cascades of Aerofoils. Note No. E.3696, British R.A.E., June 1939.

The performance of a cascade of airfoils is expressed in terms of deflection of the flow and total-pressure loss. The design of blading for an axial-flow compressor is based on a radius at which 50-percent reaction occurs. Under such conditions, Constant's empirical rule is  $\delta = 0.26 \sqrt{s/c}$ , where  $\delta$  is deviation angle,  $s$  is pitch, and  $c$  is chord. The effect of stagger is small.

135. Lieblein, Seymour: Turning-Angle Design Rules for Constant-Thickness Circular-Arc Inlet Guide Vanes in Axial Annular Flow. NACA TN 2179, 1950.

A survey of data taken from several axial-flow-compressor inlet guide vanes leads to the establishment of a linear relation between the vane camber and air-turning angles. The guide vanes were circular-arc, constant-thickness-section airfoils operating at zero angle of incidence for a range of solidities from 1.4 to 1.7. The inlet Mach number was 0.3, the convergent annular area ratio varied from 0.86 to 0.95, and the turning angles varied from  $12^\circ$  to  $40^\circ$ . Cascade test data were obtained from survey 120.

The linear relation of air turning angle  $\theta$  with vane camber angle  $\phi$  at zero incidence is given for turning angles from  $10^\circ$  to  $41^\circ$  by  $\theta = 0.985\phi - 9.7$ . By correcting the turning-angle data to constant axial velocity on the basis of the inlet velocity and assuming constant circulation, a design rule applicable for a wide range of axial-velocity ratio across the vanes is obtained:

$$\theta = -0.0087\phi^2 + 1.492\phi - 13.87$$

For vanes at incidence angles  $\gamma$  unequal to zero, the turning angles for both variations can be approximated for incidence angles from  $-10^\circ$  to  $5^\circ$  by using  $d\theta/d\gamma = 0.88$  for camber angles between  $25^\circ$  and  $40^\circ$ .

136. Mortarino, C.: *Esperimenti su una schiera di polettaggi per compressore*. L'Aerotecnica, t. XXX, no. 2, Apr. 15, 1950, pp. 59-72. (Experiments on a Cascade of Blades for Compressors.)

The experimental results of measurements made on a cascade of blades are presented. The experiments had three objectives: (1) to describe devices and procedures determining the characteristics of blade cascades as a basis for axial-flow-compressor design, (2) to compare experimental results with theory, and (3) to obtain data for the design of nose sections and data concerning energy losses, the influence of boundary layers, and tip clearance.

The experiments were conducted in the incompressible-flow range at Mach numbers of about 0.17 and Reynolds numbers of about 400,000 for a range of stagger and incidence angles. The aerodynamic characteristics were obtained by determining the forces from the measured pressure distribution around the profile and by applying momentum considerations to measurements of velocities upstream and downstream. Flows on the surface were traced by the use of droplets of oil paint.

The experiments showed clearly the three-dimensional nature of the flow. Theoretical and experimental agreement were obtained when the measurements were corrected for the convergence of the streamlines (Ferrari correction, surveys 2 and 3). The tip-clearance tests indicated that small tip clearances are desirable (see survey 230).

137. Mortarino, C.: *Esperimenti su alette in schiera per funzionamento a turbina*. L'Aerotecnica, t. XXXII, no. 4, Aug. 1952, pp. 192-205. (Experiments on Blade Cascades for Turbines.)

Tests for further experimental investigation of the Ferrari correction for streamline convergence (see survey 136) were conducted in the incompressible-flow range with a Mach number 0.17, Reynolds numbers about 400,000, and gap-chord ratio of 0.7. Flow surveys and flow-visualization methods were used. Many diagrams are presented of lift and drag coefficients and turning angles.

Comparisons with the Ferrari formulas indicate that there is good agreement between the calculated and experimental slopes of lift coefficient curves. Curves of the drag coefficient against lift emphasize the great difference in performance between blades in cascades and as isolated airfoils. For the tests conducted on cascades, the profile peak efficiency point was the point of maximum lift.

138. Briggs, William B.: *Effect of Mach Number on the Flow and Application of Compressibility Corrections in a Two-Dimensional Subsonic-Transonic Compressor Cascade Having Varied Porous-Wall Suction at the Blade Tips*. NACA TN 2649, 1952.

Tests were conducted on NACA 65-(12)-10 blades using the porous wall techniques of survey 207 and extending the application into the compressible-flow range. The inlet flow Mach numbers ranged from 0.12 to 0.89. Boundary-layer suction slots were provided on the walls ahead of the cascade, and porous walls were used near the blade tips. The boundary-layer removal was controlled to satisfy (as measured experimentally) the two-dimensional continuity relation upstream and downstream of the cascade. The experimental data presented include variations of turning angle, wakes, pressure distribution, and static pressure with Mach number, with and without boundary-layer control.

The wall boundary-layer effects can be removed with suction, and good centerline agreement can be obtained experimentally in the stationary cascade with theoretical values and values obtained in rotating cascades. Where suction was applied to a cascade of blades with an aspect ratio of 4 to 1, little effect was noted on the wake at midspan. However, large effects were noted on the secondary flows, and the total-pressure loss measured at midspan was somewhat reduced. The use of suction stabilized the location of the stagnation point and decreased the shift of the peak pressure coefficient as the Mach numbers increased.

For turning-angle data alone, high-speed tests or suction are not needed if configurations with 4 to 1 blade aspect ratio are used. When attempts were made to correlate pressure-distribution variations with Mach number, the low-speed data could be correlated only fairly well with high-speed data by means of various correction methods. The compressibility correction methods used were the Prandtl-Glauert rule, the Kármán-Tsien rule, and the author's vector mean-velocity contraction coefficient. No consistently good agreement was obtained for any of the rules.

139. Prince, David C., Jr.: Performance of a Cascade Designed for Prescribed Loading. Preprint No. 52-SA-40, A.S.M.E., 1952.

Correction factors applied to potential-flow design methods were evaluated in order to correlate the performance of cascades in real and ideal fluids for axial-flow-compressor blade design. The tests were conducted at a Mach number of about 0.1 and a Reynolds number of about 190,000. Boundary-layer suction was applied, and spoilers were used to provide turbulent boundary layers. The cascade was designed for a flat pressure-distribution curve on the blade suction surfaces.

However, the flat pressure-distribution curve was not achieved. Many boundary-layer and secondary-flow problems were incurred and largely unresolved. (Even under good conditions the real flows at low Reynolds numbers did not approach theoretical.) With increasing angle of attack, the experimental-flow lift-coefficient deficiency and the downstream angle deviation increased when compared with the theoretical potential flow. Fair correlation was obtained between the pressure distributions. There was a high ratio of real-fluid to potential-flow circulation.

140. de Haller, P.: L'influence de l'épaisseur du profil sur les caractéristiques de grilles. Bull. Tech. de La Suisse Romande, t. 79, no. 9-10, May 16, 1953. (The Influence of the Profile Thickness on the Characteristics of the Cascade.)

The variations in blade profile thickness account largely for the discrepancies between various approximation theories and the experimental values obtained for airfoil cascade parameters. Friction and boundary-layer effects were not included in the theory developed, as they do not account for the discrepancies.

141. de Haller, P.: Das Verhalten von Tragflügelgittern in Axialverdichtern und in Windkanal. Brennstoff-Wärme-Kraft, Bd. 5, Heft 10, Oct. 1953, pp. 333-336. (The Behavior of Aerofoil Grids in Axial Compressors and in a Wind Tunnel.)

Data obtained from two-dimensional grids are applied to axial-flow-compressor design, and the data provide reliable information, if no lateral contraction of the flow occurs through the grid. Many fine compressors and turbines have been designed on the basis of two-dimensional cascade data, even for flow velocities up to Mach numbers of 0.84.

142. Stanitz, John D.: Effect of Blade-Thickness Taper on Axial-Velocity Distribution at the Leading Edge of an Entrance Rotor-Blade Row with Axial Inlet, and the Influence of This Distribution on Alinement of the Rotor Blade for Zero Angle of Attack. NACA TN 2986, 1953.

A nonuniform axial-velocity distribution may be the result of many factors, some of which are as follows: (1) boundary layer on the hub and casing, (2) curvature of the hub and casing, (3) blade curvature, (4) radial variations in angle of attack, and (5) blade taper (radial variation of blade thickness). This report investigates only the last factor. The effect of blade-thickness taper on the inlet axial-velocity distribution of an entrance rotor blade row with axial inlet flow is studied. The investigation includes compressible and incompressible nonviscous flows into an entrance rotor blade row with tapered blades and a two-dimensional cascade, respectively.

For the configuration at hand, blade taper had a large effect on the inlet deviation angle. The effect of compressibility was small, except at the hub, and the upstream relative velocity also had little effect.

143. Schlichting, Herman: Problems and Results of Investigations on Cascade Flow. Jour. Aero. Sci., vol. 21, no. 3, Mar. 1954, pp. 163-178.

A summary of theoretical and experimental research results on the flow through two-dimensional cascades is presented. Solutions of the direct and inverse problems of two-dimensional incompressible flow are obtained by a method of singularities. The subsonic compressible solutions are obtained by an extension of the Prandtl-Glauert rule. Using boundary-layer suction slots to keep the flows two-dimensional, experimental checks are provided for the theoretical results.

Annular cascades of untwisted blades and various hub-tip radius ratios were likewise investigated and results compared with two-dimensional tests. For cylindrical-walled cascades, the effects of radial divergence (variation of solidity with radius) on the pressure distributions and local loss coefficients are extremely small. In this regard, the two-dimensional viscous flow through cascades was treated by applying boundary-layer theory to obtain theoretical data for the loss coefficients.

### C. ROTATING BLADE ROWS

144. Weske, John R., and Marble, Frank E.: Characteristics of Airfoils in a Cylindrical Axial-Flow Grid. Jour. Aero. Sci., vol. 10, no. 8, Oct. 1943, pp. 289-294.

An experimental investigation was conducted upon rotating cylindrical axial-flow cascades. Measurements were taken of the pressure distribution at the airfoil mid-sections for several blade spacing and angular settings and various operating conditions. The data were used to determine the blade lift characteristics. The possibilities of supplementing the pressure-distribution measurements by wake traverses in order to calculate the profile drag were explored.

For blades with a given twist, constant circulation spanwise may be maintained at only one particular value of mid-section lift coefficient. The following results were obtained for small variations of the mid-section lift coefficient: (1) The lift characteristics at the mid-sections of blades of large pitch-chord ratios compare well with the lift characteristics (e.g., with respect to the angle of attack for zero lift and the lift-curve slope) of the same airfoil section in indefinite flow and infinite aspect ratio. (2) The maximum lift coefficient of a section in a widely spaced grid is larger than the maximum lift coefficient in infinite flow in a wind tunnel. This effect is perhaps due to centrifugal action on the blade boundary layer. (3) The angle of attack for zero lift increases as the pitch-chord ratio decreases. (4) The slope of the lift curve increases as the pitch-chord ratio decreases.

The last two results are perhaps attributable to the interference of neighboring blades due to circulation and blade-thickness effects. Because skin friction was neglected, the functional relation obtained between the lift coefficient and the angle of attack is only a first approximation. The wake traverses can be used to supplement the pressure-distribution measurements, particularly with regard to profile drag, if the flow surveys include measurements of flow direction and velocity.

145. Marcinowski, H.: The Significance of the Measured Lattice Characteristics for Calculation and Design of Axial Flow Turbines and Compressors. (Vol. 13 of ser. of articles on Compressor and Fan Design written by German engineers, coordinated by BUSHIPS 338, Navy Dept. (Washington, D.C.), May 1946.)

Two-dimensional flow through cascades was investigated to compare the maximum lift coefficient and the slopes of curves of lift coefficient against angle of attack and others for isolated and cascade airfoils. Loss measurements were taken and loss coefficients are discussed. Rotor throttling curves and loss coefficients are presented and discussed in terms of the choices of parameters (i.e., profile, camber, stagger angle, etc.) involved in rotor blade design. The discussion is qualitative and inconclusive.

146. Davis, Hunt: Use of Cascade Tests, Airfoil Tests, and Single Stage Tests for Determination of Axial-Flow Compressor Design. BUSHIPS, Navy Dept. (Washington, D.C.), June 1946.

The performance data obtained by cascade tests are limited in value because of the differences between stationary and rotating conditions. Single-airfoil and single-stage tests neglect the mutual interference effects of neighboring airfoils. Neither the turbulence factors nor the boundary-layer effects at the blade tips are taken into account. Nevertheless, each of the three tests, cascade, airfoil, and single-stage, gives some useful data and design information.

147. Runckel, J. F., and Davey, R. S.: Pressure-Distribution Measurements on the Rotating Blades of a Single-Stage Axial-Flow Compressor. NACA TN 1189, 1947.

Pressure-distributions were measured about the mean-radius section of rotating blades at a blade Mach number of 0.35 with an NACA multicell rotating pressure-transfer device. The lift-curve slope was found to be lower than the values estimated theoretically from comparable two-dimensional cascades. The need for and use of cascade test data to determine blade-angle settings are shown.

Stalling of the flow was found to originate at the root and tip sections because of casing boundary layers, improper blade twist, and large clearances. The stalling occurred sooner (lower outlet pressures and higher weight flows) than expected from isolated-airfoil maximum-lift data.

148. Pochobradsky, B.: Effect of Centrifugal Force in Axial-Flow Turbines. Engineering, vol. 163, no. 4234, Mar. 21, 1947, pp. 205-207.

Conditions are established for obtaining constant mass flow per unit area at the exit of turbine-nozzle diaphragms. The losses are assumed small, and the radial increments of pressure due to centrifugal force and compressibility are taken into account. Specific examples are given of the radial variations of exit flow angles. Free-vortex flow was found to require a radial shift of streamlines.

149. Bogdonoff, Seymour M., and Herrig, L. Joseph: Performance of Axial-Flow Fan and Compressor Blades Designed for High Loadings. NACA TN 1201, 1947.

Four rotor blades of free-vortex flow design with design pitch section lift coefficient from 0.31 to 0.99 were tested. The blades were designed for higher peak loading than were in use at the time of the report. Yaw angles and pressures were surveyed. The blades gave a maximum peak efficiency of around 96 percent. Low loading caused a decided drop in peak efficiency. Leading-edge roughness caused a drop in both the efficiency (from  $2\frac{1}{2}$  to 3 percent) and pressure rise (from 11 to 15 percent) at the design point.

With solidities near 1, design lift coefficients approaching 1 can be used with high efficiencies. A maximum lift coefficient of at least 1.4 is obtainable. The measured performances were close to those obtained in survey 121.

150. Bogdonoff, Seymour M., and Hess, Eugene E.: Axial-Flow Fan and Compressor Blade Design Data at 52.5° Stagger and Further Verification of Cascade Data by Rotor Tests. NACA TN 1271, 1947.

In an attempt to increase the accuracy of existing design charts and to measure their precision, cascade and blower tests were conducted to obtain the performance of blades in a rotating configuration. The report follows survey 121. The tests are conducted on NACA 65-410, 65-810, and 65-(12)10 blower-blade sections at solidities of 1.0 and 1.5.

Cross plots show that over a range from 44° to 65°, the turning angles predicted on the basis of cascade tests are within 1/2 percent of those obtained in the rotating setup. The high loading of the NACA 65-(12)10 blades appears to be a limit beyond which cascade-tunnel data are questionable. For blades with higher loading, slight changes in tunnel adjustment may produce tunnel-wall stall and greatly change the blade performance. For these reasons, data obtained for NACA 65-(18)10 blades have been omitted.

151. Herrig, L. Joseph, and Bogdonoff, Seymour M.: Performance of an Axial-Flow Compressor Rotor Designed for a Pitch-Section Lift Coefficient of 1.20. NACA TN 1388, 1947.

Compressor blades with higher cambers than could be tested successfully in survey 122 were tested in a single-stage blower. Low-speed tests were made of the highly cambered blades in an axial-flow-compressor rotor with no guide vanes or stator blades, with uniform inlet-velocity distribution, and with little boundary layer. Under these conditions, rotor efficiencies of 96 percent were obtained for blades with a mean lift coefficient of 1.2.

In more detail, the rotating tests showed an increase in design loading to be possible, causing the pressure ratio per stage to increase while maintaining a good operating range and a 96-percent peak efficiency. Deviations of the peak efficiency from the design point indicate that optimum performance of higher camber sections would occur at lower angles of attack. At a solidity of 1.0, an average lift coefficient of 1.2 can be obtained with good efficiency, and maximum lift coefficients above 1.4 are indicated.

Extreme leading-edge roughness caused an efficiency drop of 3.5 percent and an 11-percent drop in pressure rise at design conditions.

152. Bogdonoff, Seymour M.: N.A.C.A. Cascade Data for the Blade Design of High-Performance Axial-Flow Compressor. Jour. Aero. Sci., vol. 15, no. 2, Feb. 1948, pp. 89-95.



The results of this investigation indicate that information on turning angle and design angle of attack obtained in low-speed tests can be used for high-speed design. The Mach number for critical speed turned out to be 4 to 5 percent higher than obtained by the Kármán-Tsien extrapolation. Extrapolation of low-speed rotor test results to compressor stages operating at efficiencies of 90 percent and below a critical Mach number indicates an obtainable pressure ratio of 1.4. Increased compressor performance can be obtained by use of 6-percent-thick tip-section blades instead of 10-percent-thick blades. The limiting Mach number is 3 to 4 percent higher for the 6-percent-thick blades.

153. Khalil, K. H.: Wind Tunnel Investigations on Rotating Blades of Aerofoil Section. Proc. Inst. Mech. Eng., vol. 166, 1952, pp. 419-428.

This investigation was to enable the use of static tests for rotating design. The pressure and lift characteristics of rotating blades are compared with those obtained for the same blade types at rest. Two important experimental results are obtained, the similarity in pressure diagram shapes, and the significance of a "spin" factor. A close similarity is shown in the shapes of the pressure diagrams for the rotating and static blades at the same incidence angles and radial position. However, the lift and circulation values are different, being smaller at the inner and greater at the outer radii for the rotating blades.

When the system of trailing vortices is compared for rotating and stationary cases, the difference along the span depends only on the speed of rotation. This is probably caused by added uniform vortex strength, which is a true indication of the results of rotation. Thus, test results from stationary to rotating blades in wind tunnels may be corrected by adding the effects of a vortex of rotation of uniform strength.

154. Khalil, K. H.: Rotational Effects on a Cascade of Aerofoil Blades. The Engineer (London), vol. CXCIII, no. 5030, June 20, 1952, pp. 831-834.

The optimum pitch for a cascade under rotational conditions is determined in comparison with that obtained by simple cascade tests. Pressure distributions around a blade section were obtained for a range of incidence angles by varying the components of the flow, the axial velocity (keeping the resultant velocity constant at 70 ft/sec), and the rotational speed over a range from 50 to 800 rpm. The pitch and the number of blades were varied, but the product of the pitch times the number of blades was kept constant.

When the rotating blades were compared with stationary blades having the same sections, the circulation of the rotating blades was found to be diminished. The effect of the rotation increased as the number of blades increased. With shrouding, the pressure-distribution diagrams showed changes near the leading edges rather than near the trailing edges.

155. Lieblein, Seymour: Theoretical and Experimental Analysis of One-Dimensional Compressible Flow in a Rotating Radial-Inlet Impeller Channel. NACA TN 2691, 1952.

Passage mean-flow characteristics in a radial-inlet impeller channel are obtained from a one-dimensional compressible-flow analysis. A theoretical investigation of the flow in an impeller channel with convergent-divergent area showed the critical section of the rotating channel was located upstream of the geometric throat. The effect of losses on the flow was similar to the effect obtained by reduction of the flow area. The mean-flow behavior in another radial-inlet impeller was similar to flow along a rotating radial channel, in which the effective flow area at the inlet varied with the operating point.

156. Finger, H. B.: Method of Experimentally Determining Radial Distributions of Velocity Through Axial-Flow Compressors. NACA TN 2059, 1950.

A method is presented for determining the velocity distribution downstream of a compressor blade row for given total temperature, total pressure, and relative flow angles. The results of this analysis are in good agreement with experimental results obtained on a ten-stage compressor. The predicted and measured wall static pressures give good checks.

#### D. COMPRESSOR PERFORMANCE AND CHARACTERISTICS

157. Sinnette, John T., Jr., Schey, Oscar W., and King, J. Austin: Performance of NACA Eight-Stage Axial-Flow Compressor Designed on the Basis of Airfoil Theory. NACA Rep. 758, 1943. (Supersedes NACA WR E-1.)

The development, construction, and testing of an eight-stage axial-flow compressor are described. The design theory of a typical axial-flow-compressor stage, developed directly from basic airfoil-theory fundamentals, is presented. A physical description of the eight-stage compressor discusses dimensions, construction, operation theory, test procedures, instrumentation, and precision of measurements. The design basis of each stage was the symmetric velocity diagram and radially constant axial-velocity component. The rotor was tapered to produce an increase in axial velocity from inlet to outlet.

The performance tests were first made with rotor speed as a parameter and with compressor efficiencies based on total-pressure measurements at the discharge of the last stator row. In order to permit evaluation of the compressor performance at any desired inlet-air temperature, the data were then presented in terms of a nondimensional parameter, Mach number. This proved to be the superior procedure.

Axial-flow compressors of high efficiency (87 percent at a pressure ratio of 3.42) can be designed by the proper application of airfoil theory.

158. Eckert, B., and Weinig, F.: Axial Flow Compressor Designed for 50% Reaction. BUSHIPS 338, vol. 11, May 1946.

Three 50-percent reaction axial-flow compressors were tested experimentally to check on the efficiency of compressor design calculations for certain turbojet engines. (The reaction of a stage is the fraction of the pressure rise occurring in the rotor.) The results indicate that the total pressure and the flow capacity are markedly affected by the distribution of static-pressure rise between the rotor and the stator. For given limiting Mach numbers, the total pressure developed can be increased, if the amount of reaction is limited to 50 percent. The significance is that the static-pressure rise per stage determines the number of stages required. Satisfactory agreement was obtained between theory and experiment.

Flow regulation by means of stator adjustment was attempted but was neither efficient nor worthwhile. Hub fairing was necessary to reduce tendencies toward flow separation and poor velocity distributions at the rotor inlet. The exact shape of the fairing was not critical.

159. Eckert, B.: A Collection of Compressor Test Results. BUSHIPS 338, vol. 22, Navy Dept., May 1946.

A summary of various compressor test results is presented, containing approximately 90 characteristic plots of compressor performance. The coefficients used to

plot the compressor characteristics are explained, and the methods of obtaining them from tests are described. The effects of Reynolds number and Mach number variations on compressor design are defined.

160. Eckert, B.: Recent Results on High-Pressure Axial Blowers. NACA TM 1128, 1947.

The proper aerodynamic design of inlet ducts to achieve high-efficiency operation and high pressure rise is discussed. The study includes the effects of hub shrouds, diffuser shapes, partial admissions, radial clearances, blade shapes, and roughness. Pressed sheet-metal blades were found as good as streamlined shapes.

161. Weske, J. R., Rubin, B. Z., and Gardiner F. J.: Axial Supercharger XI-2220 Engine - Theory for Design of Axial Flow Compressors. Eng. Div., Chrysler Corp., June 9, 1945.

The performance analysis and calculation procedures are presented for an axial-flow compressor of symmetric-velocity-diagram flow, which is compared with a vortex-flow design. Much larger pressure increases can be obtained for given limiting conditions by the symmetrical-velocity-diagram type. However, some slight efficiency decreases result as well, and special inlet and outlet stages are required. Computations for these stages are provided.

162. Jarre, Gianni: Un nuovo tipo di stadio di compressione per compressori assiali. L'Aerotecnica, t. XXVIII, no. 4, 1948, pp. 199-205. (A New Type Compression Stage for Axial Compressors.)

The stage performance of an axial-flow compressor is analyzed, and a simplified two-dimensional calculation procedure for multistage compressors is explained. Numerical calculations demonstrate the possibilities of reducing the number of stages required for a multistage compressor.

163. Ponomareff, A. I.: Axial-Flow Compressors for Gas Turbines. A.S.M.E. Trans., vol. 70, no. 4, May 1948, pp. 295-306.

The operating principles, characteristics, design, operation, types, characteristic curves, and gas-turbine requirements for axial-flow compressors are presented and explained.

164. Mankuta, Harry, and Guentert, Donald C.: Investigation of Performance of Single-Stage Axial-Flow Compressor Using NACA 5509-34 Blade Section. NACA RM E8F30, 1948.

The performance of a single-stage axial-flow compressor using blades with an NACA 5509-34 airfoil section was investigated. The blades were designed with a hub-tip radius ratio of 0.8 in order to correspond to the middle stages of a compressor. The stators were designed to remove the vortex rotation added by the rotor. A symmetrical velocity diagram at the rotor hub position was used. The blade setting angles were taken according to survey 119.

The performance was evaluated in terms of three different blade-loading parameters: turning angle, lift coefficient, and the ratio of the change in tangential velocity to mean axial velocity. The over-all performance results are presented (maximum total-pressure ratio, 1.262; maximum adiabatic efficiency, 0.84 at design speed; and equivalent weight flow, 10.5 lb/sec). Since instrumentation difficulties were encountered, the results are considered only qualitatively correct.

165. Burt, Jack R.: Investigation of Performance of Typical Inlet Stage of Multi-stage Axial-Flow Compressor. NACA RM E9E13, 1949.

Analyses have indicated that a compressor design based on constant total enthalpy radially with a symmetrical velocity diagram at all radii has advantages over a vortex-flow design compressor (see survey 183). Accordingly, a compressor inlet stage, designed on this basis, was tested and studied. Typical inlet-stage values were used, that is, a hub-tip radius ratio of 0.5 and an axial-velocity-component to tip-speed ratio of 0.60. The rotor and stator blades had a modified NACA 65-(12)10 profile, constant from hub to tip. Blade setting angles were calculated according to survey 119. The instrumentation was the same as for survey 164, and the data were calculated by the method of survey 121. The rotor blade performance indicates that the turning angles and the energy addition at design weight flow were low compared with design values. These results were attributed to miscalculation of the turning-angle deviations near the rotor blade tips.

Nevertheless, at off-design conditions, good weight-flow, pressure-ratio, and efficiency characteristics were observed. The high rotative speed of such an inlet stage permits higher pressure ratios in the later stages of a compressor. The use of variable-camber blades may have improved the stage performance over a wider weight-flow range. This investigation is followed by survey 172, a study to reduce the wheel speed and pressure ratio but obtain increased mass flow.

166. Martinuzzi, P. F.: Continental and American Gas-Turbine and Compressor Calculation Methods Compared. A.S.M.E. Trans., vol. 71, no. 4, May 1949, pp. 325-333.

The European methods for calculating axial-flow-compressor blading involve dimensionless velocity triangles, the characteristics in terms of a pressure coefficient, flow coefficient, profile "glide" angles, and degree of reaction. The blading types, given in terms of their velocity triangles at the pitch diameter, include axial discharge from rotor, symmetrical stages, stages with axial rotor-inlet flow, and stages with symmetrical stators. Constant axial velocity radially, constant circulation, and free-vortex flow between blade rows are assumed.

The various blading types are compared and applied.

167. Bowen, John T., Sabersky, Rolf H., and Rannie, W. Duncan: Theoretical and Experimental Investigations of Axial Flow Compressors, Pt. 2. Mech. Eng. Lab., C.I.T., July 1949. (Navy Contract N6-ORI-102, Task Order IV.)

To examine closely the validity of the design theory of survey 37, extensive and detailed flow measurements were made in the test compressor, and boundary-layer behavior and friction losses were also investigated.

The study of this proposed design theory, which uses the exit angle as a basic parameter in place of the lift coefficient and a new linearizing procedure, is conducted by comparing two blade types, vortex and wheel. The theory assumptions are classified as three main types: (1) perfect fluid assumptions, (2) assumptions concerning exit-flow angles, and (3) linearizing assumptions. Test results indicate that the theory yields accurate results in predicting the flow characteristics at design and off-design conditions, based on comparisons of the velocities and total pressure at flow conditions for which assumptions (1) and (2) are valid. Assumption (1) was valid when no part of the blade was stalled. The over-all performance was predicted accurately near the design point, which was close to the maximum-efficiency point. The free-vortex and solid-body blading showed nearly the same efficiencies, and the exit angles were insensitive to inlet-angle changes. The efficiency increased with increasing Reynolds numbers.

The investigation into the cascade boundary layers and losses disclosed information concerning the location and extent of losses encountered in a turbomachine. The minimum total-pressure loss and incidence angles for low loss can be predicted for a two-dimensional cascade. Careful measurements of blade-surface-friction losses (total-pressure losses in the cascade) were made. The total-pressure losses in the compressor cascade were twice those measured or calculated for equivalent two-dimensional cascades in steady flow. The apparent flow incidence-angle range for low losses is comparably smaller. These results are attributed to secondary flows (see surveys 230 to 234) and periodic velocity fluctuations in entrance velocity (see surveys 53 to 59, and 69).

The studies indicate that the wall boundary-layer growth was moderate in the test compressor and depended largely on the blade design (survey 231). High losses were found near the inner and outer walls. An apparent "area contraction" of the compressor flow stream may be attributed to wall and blade boundary-layer effects.

168. Howell, A. R., and Bonham, R. P.: Over-All and Stage Characteristics of Axial Flow Compressors. Proc. Inst. Mech. Eng., vol. 163, 1950, pp. 235-248.

A method is developed for deriving the characteristics of multistage axial-flow compressors from the characteristics of the individual stages for a range of operating conditions. This proposed design method enables the single-stage characteristics to be predicted, designed, and thus extended to the over-all characteristics for the required design conditions. The effects of blade and test errors on performance are discussed.

169. Erwin, John R., and Schulze, Wallace M.: Investigation of an Impulse Axial-Flow Compressor. NACA RM L9J05a, 1950.

Because of the static-pressure rise, the total-pressure coefficient obtainable in conventional axial-flow compressors is limited to a maximum of approximately 1.0 by separation losses, when the turning angles exceed  $30^\circ$ . Turning the flow without a static-pressure rise extends the limit of turning angle and makes higher pressure coefficients possible. This investigation was made to determine the possibilities of applying the constant-pressure (impulse) principle to axial-flow compressors.

Blade sections developed by cascade tests were used in the design of a rotor with  $75^\circ$  turning and a total-pressure coefficient of 2.4. Rotor tests were conducted at low speeds both with and without stator blades and without inlet-guide vanes. For example, rotor operation produced a total-pressure coefficient of 2.3 with an efficiency of 98.3 percent.

The operating behavior of the single-stage impulse compressor was similar to conventional single-stage operation. Extrapolation of the results to higher subsonic speeds indicates that the impulse compressor can produce the pressure rise per stage of a conventional compressor, but at reduced rotational speeds. There may be accompanying losses in specific mass flow and efficiency. At supersonic speeds, the impulse compressor should yield a stage pressure rise equal to the conventional supersonic axial-flow compressor and operate at reduced rotational speeds. Again, there may be losses in specific weight flow.

170. Mahoney, John J., Dugan, Paul D., Budinger, Raymond E., and Goelzer, H. Fred: Investigation of Blade-Row Flow Distributions in Axial-Flow Compressor Stage Consisting of Guide Vanes and Rotor-Blade Row. NACA RM E50G12, 1950.

A 30-inch-tip-diameter axial-flow compressor stage was investigated with and without the rotor to determine individual blade-row performance, interblade-row effects, and outer-wall boundary-layer conditions. Angle settings were made according to survey 119.

With no rotor, flow-angle discrepancies were observed at the root and tip of the guide vanes. However, the design assumption of simplified radial equilibrium was satisfied when corrected angles were considered. For the medium and high weight-flow ranges, an inlet Mach number correction is not sufficient to account for the increases in the guide-vane-exit flow angles at the hub and tip. With the rotor installed, the design assumption of simplified radial equilibrium is invalid.

Curves of the measured variations of turning angles across the rotor with angle of attack paralleled curves obtained from two-dimensional-cascade data but gave higher turning angles. Tip stall was obtained at lower angles of attack than in the two-dimensional cascade. The boundary-layer displacement thickness was small after the guide vanes and the rotor (less than 1.0 and 1.5 percent of passage height, respectively) but increased rapidly after the rotor when tip stall occurred.

171. Moses, Jason J., and Serovy, George K.: Effect of Blade-Surface Finish on Performance of a Single-Stage Axial-Flow Compressor. NACA RM E51C09, 1951.

The performance of a single-stage axial-flow compressor was investigated under conditions of varying blade-surface roughness. The blade surfaces were successively rough-machined, hand-filed, and highly polished. The compressor was operated over a range of weight flows at six equivalent tip speeds from 672 to 1092 feet per second, for relative inlet Mach numbers from 0.36 to 0.85 at the rotor mean radius, and for Reynolds numbers from 222,000 to 470,000.

Surface-finish effects were measured by the total-pressure ratio and efficiency effects observed. The surface-finish effects decreased with increasing compressor speed and with decreasing weight flows at a fixed speed. Finishing the blade surfaces beyond a point considered aerodynamically smooth (from an admissible roughness formula) will not affect compressor performance, except at operating points where blade friction losses may be an important fraction of the total losses.

172. Burt, Jack R., and Jackson, Robert J.: Performance of Single-Stage Compressor Designed on Basis of Constant Total Enthalpy with Symmetrical Velocity Diagram at all Radii and Velocity Ratio of 0.7 at Rotor Hub. NACA RM E51F06, 1951.

The basic compressor design variables were analyzed in survey 163, and it was found that good performance may be obtained from a compressor designed for constant total enthalpy radially and a symmetrical velocity diagram at all radii. A compressor of this design, with a hub-tip radius ratio of 0.5 and a ratio of axial velocity to tip speed of 0.6 upstream of the rotor hub, is investigated in survey 165. This compressor had high pressure ratios at high rotative speeds.

This investigation was conducted on a compressor designed for an axial-velocity to tip-speed ratio of 0.7 at the rotor hub to reduce wheel speeds and pressure ratios, but increase mass flows.

The results indicate a 12-percent increase in mass flow at peak pressure ratios somewhat higher than the earlier compressor, but with greatly reduced efficiency. The low solidity and high stagger angle near the rotor tip combined to produce a rapidly peaking curve of stage-element efficiency. The good efficiency range for a given speed is narrow. Near the hub, the efficiency is nearly constant over the range of weight flows investigated, assuming the streamlines there follow the hub contours.

The temperature-rise energy addition at a given speed increased from hub to tip. With change in weight flow, the greatest rate of change of energy addition likewise occurred at the tip.

173. Panetti, Modesto: Una ipotesi limite per il calcolo della caratteristica dei compressori assiali. La Termotecnica, t. 5, no. 1, 1951, pp. 5-10. (A Limit Hypothesis for the Calculation of the Characteristics of Axial Compressors.)

With the assumption that the discharge angle remains constant for the entire range of inlet angles, laws concerning the variation of lift and the pressure rise per stage as functions of incidence angles and inlet velocities are determined. These results agree well with experimental data and appear to be good approximations for cascades with solidity of approximately 1, for operation not near stall conditions. The influence of viscosity and friction losses on a pressure-volume characteristic are accounted for by assuming a reduced exit area as a result of boundary-layer growth.

174. Panetti, Modesto: Il compressore assiale sperimentale del laboratorio di aeronautica di torino. La Ricerca Sci., t. 23, no. 9, Sept. 1953, pp. 1639-1644. (The Experimental Axial Compressor at the Aeronautical Laboratory of Turin.)

The operational characteristics and design details of a free-vortex-design compressor having a symmetrical velocity diagram at the mean section (the Franco Tosi experimental axial-flow compressor) are presented. High efficiencies were obtained in the normal operating range.

175. Taylor, R. G.: Some Blade Designs for an Axial-Flow Compressor Stage. Jour. Roy. Aero. Soc., vol. 58, no. 517, Jan. 1954, pp. 61-64.

The over-all conditions in a single stage without guide vanes are discussed for three cases: (1) the static-temperature rise in the rotor and stator are equal (constant reaction with radial equilibrium), (2) the Mach number of the resultant flow is constant radially at the rotor inlet, and (3) a combination of (1) and (2).

No information is presented to indicate whether any (or all) of these designs is suitable or practical.

176. New, W. R., Redding, A. H., Saldin, H. B., and Fentress, K. O.: Basic Compressor Characteristics from Tests of a Two-Stage Axial-Flow Machine. A.S.M.E. Trans., vol. 76, no. 3, Apr. 1954, pp. 473-481.

Experimental results are presented for axial-flow-compressor stage performance over a range of Mach numbers from 0.3 to 0.9 and a range of Reynolds numbers from 50,000 to 500,000. At subsonic velocities, the tests show that the attainable pressure ratio per stage and efficiency depend on the Reynolds and Mach numbers. Many diagrams are presented to visualize these relations. The results are presented in pairs of plots showing first, a basic efficiency and stage pressure ratio as functions of velocity ratio with the Reynolds number as a parameter, and second, a Mach number correction for each plot.

177. Foster, D. V.: The Performance of the 108 Compressor Fitted with Low Stagger Free Vortex Blading. Rep. No. R.116, British N.G.T.E., June 1952.

Tests were conducted on a large three-stage compressor designed especially for detailed three-dimensional flow investigations. The main errors in the accuracy of the measurements were attributed to unsteady flow and speed fluctuation effects (see surveys 53 to 59, and 69). The test characteristics of the first set of blades were compared with the theoretical performance calculations. The tests were conducted with and without guide vanes. The following results and conclusions were obtained: (1) The compressor proved to be suitable for its designated purpose. (2) The effect

of blade position on the measured static pressure was very large. (3) The deviations at the mean diameter, smaller than those obtained with a two-dimensional flow configuration, permitted increased work capacity of the compressor. In those cases where the inlet guide vanes produced the two-dimensional outlet angles, the first-stage characteristics were different from the characteristics of the other two stages. (4) The velocity profile did not degenerate rapidly (see survey 213). This fact is attributed to good flow conditions near the blade ends. (5) High efficiencies, obtained over a wide range of mass flows, were considered to result from good velocity profiles. (6) Usual methods of calculating on- and off-design performance, stage by stage, yield predictions of performance lower than obtained in test results.

Surging and stalling are discussed briefly.

178. Kantrowitz, Arthur: The Supersonic Axial-Flow Compressor. NACA Rep. 974, 1950. (Supersedes NACA ACR L6D02.)

The advantages and disadvantages of supersonic compressors are discussed in an attempt to obtain the inherent advantages of large increases in pressure ratio per compressor stage and high mass flow at good efficiencies, thereby effecting savings in weight and size. The wave systems at the entrance regions of a cascade and the losses due to waves extending upstream in supersonic flows are studied. Methods of eliminating the waves and accompanying losses in supersonic flows and of confining the strong waves entirely within the cascade passages are discussed. The conclusions reached from this discussion and a discussion of starting conditions are that compressors operating with relative Mach numbers considerably above 1.0 are more likely to be of practical interest than those operating near relative Mach number 1.0.

Three design types are analyzed in some detail: (1) supersonic relative velocities entering and leaving the rotor having a subsonic stator, (2) supersonic relative velocities entering and decelerating through normal shock to subsonic flow in rotor, with subsonic flow in the stator, and (3) subsonic flow in the rotor, with supersonic inlet flow to stator and deceleration to subsonic flow through normal shock in the stator passages.

A compressor of the second design type, deceleration through shock in the rotor, was constructed and tested (at comparable Mach numbers in Freon-12, see survey 203). Other results and conclusions obtained are as follows: (1) The transition from subsonic to supersonic flow occurs smoothly. (2) Pressure ratios of about 1.8 and efficiencies of about 80 percent were obtained. The need for stronger blading and higher solidity was indicated by the tests. (3) The volume flow was close to theoretical values. (4) Higher efficiencies could be obtained by methods of controlling the wave formation in a specified fashion. (5) Efficient supersonic compressors can be designed of much higher stage pressure ratios and somewhat higher mass flows than attainable by subsonic compressors. (6) The starting of a supersonic compressor with subsonic axial velocities will not involve undue difficulties.

179. Loeb, W. A.: A Study of the Supersonic Axial-Flow Compressor. Jour. Appl. Mech., vol. 16, no. 1, Mar. 1949, pp. 19-26.

An analytical study is presented of two kinds of compressors, one with a normal shock in the stator entrance, the second with a normal shock in the rotor entrance. The analysis is one-dimensional and assumes idealized blade-element flow, in which radial and friction effects are ignored. Curves are drawn for the pressure ratio and efficiency as functions of a dimensionless wheel-speed parameter.

The compressor with normal shock in the stator entrance is considered the superior one. Its strongest restriction is the Mach number limitation in the rotor. The influence of limitations on blade entrance angles is discussed.



## E. COMPRESSOR VARIABLES, PARAMETERS, AND VELOCITY DIAGRAMS

180. Eckert, B.: The Influence of Physical Dimensions (Such as Hub:Tip Ratio, Clearance, Blade Shape) and Flow Conditions (Such as Reynolds Number and Mach Number) on Compressor Characteristics. Pt. A - Summary of the Results of Research on Axial Flow Compressors at the Stuttgart Research Institute for Automobiles and Engines. W.A.C. Eng. Trans. No. 22, Wright Aero. Corp. (Vol. 3 of ser. of articles on Compressor and Fan Design, written by German engineers, coordinated by Code 338, BuShips, Navy Dept. (Washington, D.C.), May 1946.)

3886 A summary of the results of research on axial-flow compressors at Stuttgart is presented, including methods for calculating axial-flow-compressor performance, means of increasing stage pressure ratios (e.g., by higher wheel speeds, manufacturing precision, and operation at higher Reynolds numbers), and influence of diameter ratio, efficiency, staging effects, and Reynolds number.

The influence of diameter ratio on the characteristics and velocity diagram of axial-flow compressors is discussed. Experimental results and some theoretical correlation are provided. The general conclusion is that increase of diameter ratio causes the attainable pressure coefficient, the throttling coefficient, and the maximum efficiency all to decrease.

The influence of radial clearance of the rotor on compressor efficiency is also shown. Both test results and theory are described. The results, which were both qualitative and inconclusive, found that efficiency decreased as tip clearance increased.

181. Eckert, B., Pflüger, F., and Weinig, F.: The Influence of the Diameter Ratio on the Characteristics Diagram of the Axial Compressor. NACA TM 1125, 1948.

The influence of diameter ratio on the efficiency and the pressure coefficient of an axial compressor was investigated. The diameter ratio was found to be very important for highly loaded axial-flow compressors. The maximum attainable pressure coefficient and efficiency are shown to be functions of diameter ratio, specific speed, and throttling coefficient. Both the maximum pressure coefficient and the optimum efficiency decrease with an increase in diameter ratio. These effects are attributed to increased drag effects due to the increased wetted surface involved.

The optimum efficiency obtainable depends on the specific rotary speed and on the mass-flow coefficient, related to inlet area. The internal adiabatic efficiency is very sensitive to the lift-drag ratio, which leads to the recommendation of optimum surface quality finish and precision of manufacture.

182. King, J. Austin, and Regan, Owen W.: Performance of NACA Eight-Stage Axial-Flow Compressor at Simulated Altitudes. NACA WR E-5, 1944. (Supersedes NACA ACR E4L21.)

The design-point performance characteristics of the NACA eight-stage axial-flow compressor and the effects of altitude on performance were determined. The compressor was tested at simulated altitudes of 50,000, 36,000, and 27,000 feet, and at rotor speeds corresponding to compressor Mach numbers of 0.80, 0.85, 0.90, and 0.95 for a range of air flows.

The design pressure ratio of 5 was obtained at a simulated altitude of 36,000 feet at an adiabatic temperature-rise efficiency of 0.83. The Reynolds number effects are manifested by a decrease in efficiency of 4 points at Mach 0.80 for an altitude increase from 27,000 to 50,000 feet.

The effect of Reynolds numbers on compressor performance was larger than generally assumed, and higher pressure ratios per stage than usual could be obtained with good efficiency.

183. Sinnette, John T., Jr.: Analysis of Effect of Basic Design Variables on Subsonic Axial-Flow-Compressor Performance. NACA Rep. 901, 1948. (Supersedes NACA RM E7D28.)

A one-dimensional compressible-flow blade element theory for axial-flow compressors was developed and applied to the analysis of the effects of basic design variables such as Mach number, blade loading, and velocity distribution on compressor performance. A graphical method useful for approximate design calculations is presented. The relations between adiabatic and polytropic efficiencies, blade-element efficiency, blade-element pressure ratio, and specific mass flow are discussed.

The uses and limitations of blade-element theory are fully discussed. The limitations arise chiefly from blade-end and wall boundary-layer secondary-flow effects. In applying blade-element theory, the use of drag-lift-ratio values obtained from cascade measurements is suggested for the middle portion of a blade, and augmented values near the blade ends. The recommendations are only qualitative. The significant factor is the drag-lift ratio, however, not just the drag. The need for secondary-flow research and three-dimensional flow investigations is pointed out in order to obtain improved compressor designs.

The blade-element-theory approach is applied to the possibilities of improving compressor performance by considering the velocity distributions carefully. Advantages of the free-vortex design are its simplicity and the high accuracy with which the flows can be calculated. Its disadvantages arise because the inlet Mach number is well below the limiting value for most of the blade length, resulting in low stage pressure ratios and an increase in relative velocity across the rotor hub.

The symmetrical velocity diagram with constant total enthalpy at all radii eliminates these difficulties; and, in addition, the specific mass flow is increased over the free-vortex design. The inlet stage pressure ratio is higher for the symmetrical-velocity-diagram compressor; and the pressure ratios obtainable in later stages are also higher, because higher rotor speeds are possible. One disadvantage of a compressor designed for a symmetrical velocity diagram is the very low axial velocities near the tip, especially after the rotor, at low mass flows.

A velocity diagram somewhere between free-vortex and symmetrical velocity with small enthalpy variation radially is suggested as a good design compromise.

The blade-element theory is applied to an analysis of the complicated interactions among variations of axial velocity from stage to stage, rotor speed, maximum pressure ratios obtainable per stage, increase in hub and/or tip diameter axially, tip-clearance losses, blade chord size, Reynolds number effects, and boundary-layer control in the diffuser.

The analysis indicates that the effect of polytropic efficiency on some flow relations across a blade row is of the same order of magnitude as Mach number effects.

The importance of the relative Mach number on obtainable pressure ratios is shown. In order to maximize the pressure ratios, design for near limiting Mach number values on all blade elements is desirable. For a fixed inlet Mach number, increases in the ratio of mean whirl velocity to axial velocity in the high-efficiency range of the ratio (i.e., near 1) lead to rises in obtainable pressure ratios and decreases in specific mass flow.

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The velocity distribution in the inlet stage is important in compressors with fixed Mach number limitations, because the inlet stage limits the mass flow and the rotor speed, which in turn limit the pressure ratios obtainable in the rearward stages.

Adjustable stator blades are found to be effective in extending the high-efficiency range of axial-flow compressors.

184. Wattendorf, Frank L.: Mach Number Limitations on Pressure Rise in Axial Compressor Design. Eng. Div. Memo. Rep. ENG-51-9037-1-3, Army Air Forces, Materiel Command, Feb. 25, 1944.

This analysis indicates that, when the blade-inlet velocity of an axial-flow compressor is limited by local sonic Mach numbers, the maximum pressure rise per stage is obtained by a combination of high wheel speeds and prerotation in the same direction as the wheel rotation.

185. Wattendorf, Frank L.: Factors Influencing the Utilization of High Mass Flow in Axial Flow Compressors. Proc. Seventh Int. Cong. Appl. Mech., vol. II, pt. II, 1948, pp. 398-407.

Axial-flow-compressor design limitations for obtaining high mass flows are discussed. The relations between stage pressure ratio or temperature rise, the influence of hub-tip radius ratio, the influence of inlet axial Mach number on temperature rise, and the mass flow are studied for several cases. A single-stage study indicates a qualitative method of comparing compressor types.

Several conclusions are reached: (1) Increased mass flow per unit frontal area can be obtained by use of more favorable combinations of mass flow and pressure ratio per stage. These, in turn, may result from deviation from limiting assumptions, such as constant work radially and rigid body rotation. It is suggested, for an example, that flows with a variable energy input from hub to tip have more favorable characteristics than either free-vortex or solid-body flows. (2) Allowing the tangential component of velocity to vary at a rate less than free vortex permits smaller hub-tip radius ratios and greater mass flows. (3) The temperature rise at the hub will be less than at the tip.

186. Wallner, Lewis E., and Fleming, William A.: Reynolds Number Effect on Axial-Flow Compressor Performance. NACA RM E9G11, 1949.

The inlet Reynolds number effect on the performance of an axial-flow compressor was investigated. A reduction in the inlet Reynolds number from 205,000 to 65,000 decreased compressor efficiency, at constant pressure ratio, by 5 points, and the corrected mass flow 2 to 5 percent, at each compressor Mach number. Tests on two additional compressors of different design types gave the same results.

The rate of change of efficiency with Reynolds number was most pronounced at low Reynolds numbers.

187. Wu, Chung-Hua, Sinnette, John T., Jr., and Forrette, Robert E.: Theoretical Effect of Inlet Hub-Tip-Radius Ratio and Design Specific Mass Flow on Design Performance of Axial-Flow Compressor. NACA TN 2068, 1950.

An analysis of axial-flow compressors was conducted to obtain the variations of the maximum allowable rotor-tip speed, pressure ratio, and power input per unit frontal area with design mass flow per unit frontal area and inlet hub-tip radius ratio. The theoretical compressor design had constant work input radially to the

rotor, symmetrical velocity diagrams at all radii, a rotor speed giving a maximum Mach number of 0.8 in the first stage, a variable hub radius giving a maximum Mach number of 0.8 in all succeeding stages, and a flow-turning limitation based on the work of survey 31 with a blade solidity of 1.5 at the hub. The constant-enthalpy and symmetrical-velocity-diagram compressor was chosen because it yields higher pressure ratios per stage, higher specific mass flows, and higher design rotor-tip speed than the free-vortex type (see survey 183). The maximum Mach number of 0.8 was chosen because it enables a high pressure rise through the stages without resulting necessarily in appreciable efficiency losses. The modified turning limitation of survey 31 was chosen as conservative and convenient.

A polytropic efficiency of 0.9 was assumed at the mean radius, and a range of mass flows was investigated at each hub-tip radius ratio. The number of stages chosen for each inlet radius ratio was the same as the number required to give an exit hub-tip radius ratio close to 0.9. In the calculations, simplified radial equilibrium, compressibility, and blade-element flow were assumed. The streamline curvature radially was neglected.

The results of the analysis are as follows: At a given inlet hub-tip radius ratio, the design rotor-tip speed and pressure ratio per stage decreased with increasing design mass flow per unit frontal area. For decreasing inlet hub-tip radius ratio, the maximum pressure ratio per stage and rotor speed decreased. For decreasing hub-tip radius ratio, the over-all pressure ratio, specific power input, and design specific mass flow increased for most of the range covered, with limitations of constant maximum Mach number to 0.8 for all stages and exit hub-tip radius ratio 0.9. The specific power input per stage was nearly constant, varying only slightly with the hub-tip radius ratio.

188. Schulze, Wallace M., Erwin, John R., and Westphal, Willard R.: Investigation of an Impulse Axial-Flow Compressor Rotor over a Range of Blade Angles. NACA RM L50F27a, 1950.

This investigation follows survey 169 which found that, by turning the flow without static-pressure rise, the turning-angle limit of an impulse compressor could be extended over the turning-angle limit for conventional compressors, and higher pressure coefficients could be obtained. However, the specific mass flow and efficiency might be reduced. In the present study, the performance characteristics of the 75° turning impulse compressor were investigated over a wide range of blade setting angles and specific mass flows. The results of the investigation disclose that the design of an impulse axial-flow compressor is not as exacting as had been supposed. The compressor can operate over a wide no-surge range; and, with a reasonable static-pressure rise, it can efficiently produce a very high total-pressure-rise coefficient.

189. Savage, Melvyn, and Westphal, Willard R.: Analysis of the Effects of Design Pressure Ratio Per Stage and Off-Design Efficiency on the Operating Range of Multistage Axial-Flow Compressors. NACA TN 2248, 1950.

The effects of the off-design efficiency characteristics and design stage pressure ratios on the operating range and over-all efficiency of an axial-flow compressor are studied. The results indicate that compressors with high stage pressure ratios have higher over-all off-design efficiencies and wider operating ranges than compressors with low stage pressure ratios, if the blade-row efficiency curves for the two cases are considered to be similar.

190. Bogdonoff, Seymour M.: The Performance of Axial Flow Compressors as Affected by Single Stage Characteristics. Rep. No. 155, Aero. Eng. Lab., Princeton Univ., Oct. 1, 1949. (See also Jour. Aero. Sci., vol. 18, no. 5, May 1951, pp. 319-328.)

Calculations are presented for the off-design behavior of three compressors with 6, 8, and 13 stages, all designed for the same performance characteristics. That is, each was designed to produce an over-all pressure ratio of 3.2, the same mass flow, the same mean blade speed of 648 feet per second, the same axial velocity of 450 feet per second, and symmetrical velocity diagram at all radii. The different numbers of stages to do the same work were obtained by altering the change in the tangential velocity component across the blade rows. This means the stage loading was higher as the number of stages decreased. Accordingly, the air inlet and outlet angles were different for each of the three compressors.

The blade performance data and efficiencies, on which the compressor calculations were predicted, were obtained from cascade and isolated-airfoil tests. Three-dimensional and stage interference effects were neglected. The single-stage variables studied were blade loading, or pressure rise per stage, and efficiency variations with mass flow.

At design speed, the six-stage compressor had the widest mass-flow range but the smallest (flattest) pressure-ratio range and efficiency variation with mass flow, because the change in percentage in developed pressure ratio or axial velocity is smallest for highly loaded blades.

Multistage compressor stalling is discussed. It is proposed that the sudden sharp stall phenomenon in a multistage compressor might be eliminated by compelling a compressor to stall at its rear stages first.

191. Quenzer, H., und Schwartz, G.: Aerodynamische Berechnungsmethoden für hochbelastete Axialverdichter. Schweizerische Bauzeit., Jahrg. 69, Nr. 31, Aug. 1951, pp. 432-435; 462-465. (Aerodynamic Calculation Methods for High-Power Axial Compressors.)

A survey was made of compressor design to study the limitations on delivery pressure in a compressor. The study included the degree of reaction, its radial variation and optimum values, compressibility, thickness ratio, leading- and trailing-edge angles, and spanwise variation of circulation in guide vanes, when used. Simple radial equilibrium was assumed.

Limitations imposed on the compressor Mach number limit the delivery pressure. If we define  $M_u$  as  $\frac{\text{tip speed}}{\text{velocity of sound locally}}$ , then, when  $M_u$  is less than 0.7, the rotor limits the delivery pressure. For  $M_u$  greater than 0.7, the delivery pressure is limited by conditions in the stator, in the absence of guide vanes. The limits can be raised by use of guide vanes or appropriately designed inducers.

192. Voit, Charles H., and Thomson, Arthur R.: An Analytical Investigation Using Aerodynamic Limitations of Several Designs of High Stage Pressure Ratio Multistage Compressors. NACA TN 2589, 1951.

The effects of the principal design variables on the stagewise design-point characteristics of axial-flow compressors having high average stage pressure ratios are analyzed to determine the closeness of commercial compressors for turboprop and turbojet engines to the practical limit of stage total-pressure ratios. The weight flow, total-pressure rise, and tip speed are calculated for stages having limitations on Mach number, deflection, coefficient of lift, and ratio of static-pressure rise across a blade row to the entering axial-dynamic head. Two compressor design types are compared, both for constant work radially. The first type has wheel-type flow entering the rotor and vortex flow added in the rotor. The radial position of the symmetrical-velocity-diagram point is carefully chosen. The second type is the

compressor with symmetrical velocity diagram at all radii. Provided the radial location of the symmetrical-velocity-diagram point is properly chosen, the first type is the better compressor.

The simplified analysis is then extended to multistage compressors with typical stages similar to the first type. It is found that the relative inlet flow angles to the rotor tip must be chosen carefully (i.e., lower in later stages) in order to get high pressure ratio per stage. Limiting the static-pressure rise (by increasing the axial velocity stagewise) in order to avoid boundary-layer troubles has a marked effect. Comparisons made with compressors in which the best angle (as prescribed in this report) for each group of stages (inlet, intermediate, or exit) is not chosen show the advantages of the careful choice. For given limiting conditions, the average stage total pressure was affected only slightly by deviations of the axial velocity from the optimum. However, it was considerably reduced by deviations of the relative inlet-air angle from the optimum.

193. Savage, Melvyn, and Beatty, Loren A.: A Technique Applicable to the Aerodynamic Design of Inducer-Type Multistage Axial-Flow Compressors. NACA TN 2598, 1952.

This study is one of a series of investigations of compressor designs in comparison with the simple free-vortex type. The usefulness of solid body flow was demonstrated in survey 101. Surveys 37 and 167 showed that the solid body flow compressor can be designed with efficiencies equal to those of the free-vortex type. An analysis was made comparing the wheel of the solid body type to the symmetrical velocity diagram at all radii compressors (survey 187), and the latter were found better. Another velocity diagram type, which inherently could equal or surpass the symmetrical velocity diagram at all radii types, was investigated in survey 192.

This report proposes a compressor first stage having the radial total-temperature distribution required to make the second stage inlet axial-velocity component constant radially. This, it is hoped, will reduce the axial velocities and permit higher rotational speeds. The remaining stages have constant work input radially, solid-body average tangential velocities, and radially constant inlet axial velocities.

Simplified general three-dimensional compressible-flow equations are derived which enable the axial-velocity distribution at any station to be calculated. The equations are applied to inducer compressor design to effect the desired stage characteristics. Preliminary design calculations are presented for a range of inlet hub-tip radius ratios, mass flows per unit frontal area, and stage pressure ratios.

The over-all compressor trends are as follows: (1) Average pressure ratios of 1.28 to 1.38 per stage can be attained for the range of inlet hub-tip radius ratios from 0.4 to 0.6, and at weight flows from 20 to 32.5 pounds per second per square foot of inlet frontal area. (2) The design rotational speed increases as the specific weight flow drops, at constant inlet hub-tip radius ratio. (3) The specific weight flow increases as the hub-tip radius ratio decreases, for constant speed. (4) The radial flow increases with decrease in specific weight flow at constant inlet hub-tip ratio. (5) The radial flow is sharply reduced as the inlet hub-tip ratio goes from 0.5 to 0.6, at constant weight flow. (6) Large increases in radial flow and over-all total-pressure ratio are obtained from small increments in design rotational speed. (7) The stage inlet-air angles increase stagewise. (8) The stage inlet-air angles increase as the specific weight flow decreases at each inlet hub-tip ratio.

The trends for stages following the inducer stage are as follows: (1) The amount of radial flow decreases with increasing hub-tip ratio at constant power-input ratio. (2) The radial flow increases with power-input ratio at constant hub-tip

ratio. (3) The radial flow decreases with increased hub-tip ratio, power-input ratio, and hub inlet-air angle at constant hub turning angle and tip rotational-speed ratio. (4) The power-input ratio, radial flow, and hub inlet-air angle all increase with the tip rotational-speed ratio at constant hub turning angle and hub-tip ratio.

194. Maillet, E.: Sur le perfectionnement des compresseurs et turbines. Bull. Assoc. Tech. Maritime et Aéronautique, no. 51, 1952, pp. 465-490; discussion, pp. 491-493. (Steps for the Perfection of Compressors and Turbines.)

The characteristic parameters of axial-, radial-, and mixed-flow compressors are discussed. Turbojet and turboprop units are compared. For axial-flow compressors, the manner in which cascades of different blade profiles, solidity, stagger, and pitch affect the adaptability of particular compressor types to particular duties and the effects of limiting Mach numbers, tip clearance, and boundary layers are discussed.

195. Jarre, Gianni: Compressori assiali di elevato rendimento. Giornata della Sci., 1952, pp. 39-44. (High Efficiency Axial Flow Compressors.)

The classification criteria of all types of axial compressors are presented. An analysis of the factors affecting efficiency leads to the conclusion that high efficiencies are difficult to obtain with high compression ratios per stage. The technical solution will require many compromises.

196. Naylor, V. D.: Propeller Parameters and the Axial Compressor. Aircraft Eng., vol. XXV, no. 293, July 1953, pp. 190-193.

The chief propeller parameters are discussed, and their relation to axial-compressor parameters are established. The thrust coefficient  $k_T$ , advance ratio  $J$ , and pitch ratio  $p/D$  are associated with  $p' = \frac{P}{\rho V^2/2}$ ,  $v/u$ , and the stagger angle, respectively, of compressors. Surging criteria from propeller theory are applied to the compressor case. Experimental plots for thrust coefficient and torque coefficient  $k_Q$  against advance ratio for values of  $p/D$  can be found in terms of compressor coefficients as well. Two numerical examples are given. While the propeller parameters do not explain the mechanism of airfoil stalling, they do indicate surge regions to avoid in terms of the propeller coefficients.

#### F. COMPRESSOR BOUNDARY-LAYER EFFECTS

197. Burgers, J. M.: Some Considerations on the Development of Boundary Layers in the Case of Flows Having a Rotational Component. Nederl., Akad. van Wetenschappen, vol. XLIV, nos. 1-5, 1941, pp. 12-25.

The effects of main-flow circumferential velocities and wall rotation on a viscous boundary layer are discussed. The tangential-velocity component of the main stream acts as a retarding force when the wall radius is increasing but aids in the transition to turbulent boundary layer. Rotation of the inner wall has the effect of driving the boundary-layer fluid toward sections of larger diameter and thus reducing the chances of separation.

The flow of the boundary-layer air along rotating blades, under the influence of centrifugal force and radial pressure gradients, leads to the conclusion that there is a marked tendency toward centrifugal flow in the boundary layers along the blade suction surfaces and a less significant tendency on the pressure side. Flow of the boundary layer upstream of the blades is discussed.

198. Weske, J. R.: Investigation of Blade Characteristics. Trans. A.S.M.E. vol. 66, no. 5, July 1944, pp. 413-418; discussion, pp. 419-420.

The flow through the blade rows of turbines or axial-flow compressors is obtained on the basis of airfoil theory. A correlation is established between the aerodynamic characteristics of the blade elements and the performance characteristics of the blade rows. Test results obtained on a low-solidity rotating grid indicate that the lift-coefficient range for the root and mean sections is greater than the corresponding range for fixed blades. The maximum lift coefficient is greater than that obtained from corresponding wind-tunnel data for compressors and turbines.

Experimental investigations of radial boundary-layer flows show that the blade boundary flows radially outward, when the blade rotative speed is greater than the main-stream tangential velocity component. Such boundary-layer and wake displacement outward by centrifugal force improves the lift characteristics and reduces the profile drag of the root section. When the blade rotative speed is less than the main-stream tangential velocity component, the boundary layer flows inward.

Performance curves of blade sections, based on pressure-distribution measurements on rotating blade rows, disclose a lower efficiency and higher stall limit than were obtained from airfoil data in the wind tunnel.

199. Weske, John R.: An Investigation of the Aerodynamic Characteristics of a Rotating Axial-Flow-Blade Grid. NACA TN 1128, 1947.

Pressure-distribution measurements and pitot-tube and hot-wire surveys were made to investigate the characteristics of rotating axial-flow blade rows. The radial displacement outward of the boundary layer delayed root stall and precipitated early tip stall. Boundary-layer interactions along the casing, hub, and blade surfaces and the radial shifts of the wakes are discussed.

The following conclusions are presented: (1) The assumption of an infinite aspect ratio in two-dimensional flow theories is inadequate for both blade rows with solidities of approximately 1 and thickness ratios of the present blade row (see survey 205). (2) The stalling points of the outer and middle blade sections are related, indicating that stalling spreads inward from the outer section. (3) For the rotating blade row, the root section did not stall even at very high values of lift coefficient at which the tip stalled. It is concluded that rotating decelerating blade rows can be designed for higher loading at the roots than current practice (1947). (4) Because of the low value of the effective aspect ratio, the range of minimum drag coefficient is small. The maximum lift-drag ratio occurred at less than design lift coefficient. Therefore, profiles with more camber than the present test section should be chosen for the root and midspan sections. (5) The radial boundary-layer displacement and the wakes are major factors controlling effective aspect ratio, and perhaps causing tip separation. Boundary-layer removal at the tip is suggested.

200. Boxer, Emanuel: Influence of Wall Boundary Layer Upon the Performance of an Axial-Flow Fan Rotor. NACA TN 2291, 1951.

Spoilers were used to obtain and evaluate the influence of six boundary-layer configurations on the performance, at low Mach numbers, of a single axial-flow fan designed with no allowance for boundary layers.

The drop in peak efficiency was small,  $2\frac{1}{2}$  percent, even when the disturbed flow region was greater than half the duct width. It is suggested that the loss in efficiency might be reduced by decreasing the blade pitch angle in the boundary-layer region to conform with the upstream velocity profile.



201. Alsworth, Charles C., and Iura, Toru: Theoretical and Experimental Investigations of Axial Flow Compressors. Pt. 3 - Progress Report on Loss Measurements in Vortex Blading. Mech. Eng. Lab., C.I.T., July 1951. (Navy Contract N6-ORI-102, Task Order IV.)

The results are presented of detailed measurements of the flow patterns in a single-stage axial-flow compressor with free-vortex blading. The measurements of flows through the rotor indicate that the perfect fluid design theory of survey 37 is adequate for the central 80 to 90 percent of the blading. The leaving angles were predicted quite closely, even in the stalled region. Profile losses were the same as for a similar two-dimensional cascade at moderate incidence angles, but the compressor configuration did stall at lower incidence angles.

In the stator, the loss observed near the blade ends was larger than could have been expected purely on the basis of viscosity and real flow effects on the walls (see surveys 230 and 231).

202. Wu, Chung-Hua: Survey of Available Information on Internal Flow Losses Through Axial Turbomachines. NACA RM E50J13, 1951.

The information on internal flow losses in turbomachines is briefly surveyed. Both nonviscous and viscous flow losses are discussed, and a short bibliography is included.

#### G. COMPRESSOR TESTING TECHNIQUES AND INSTRUMENTS

203. Weinig, F.: Testing of Multistage Compressors with Heavy Gas. Tech. Rep. F-TR-2143-ND, Air Materiel Command, Wright Field, Dayton (Ohio), June 1947.

Use of a suitable heavy gas such as Freon instead of air as a flow medium enables tests to be conducted on models that are geometrically similar to the full-scale air-flow configurations, but which are smaller, have lower tip speeds, smaller temperature rises, and are easier to construct. The results would apply fully and directly to the full-scale compressors, if it were possible to maintain geometric similarity, the same Reynolds and Mach numbers, and the same delivery pressure in all stages. The last two conditions can be met at two stations only, because of differences in the adiabatic exponents of Freon and air. The inlet of the first stage and the exit of the last stage are commonly chosen. The discrepancies at locations between those stations grow larger with increases in delivery pressure.

Only 1.4 percent of the full-scale power consumption was required for a 1/25-scale-model Freon compressor.

204. Huber, Paul W., and Kantrowitz, Arthur: A Device for Measuring Sonic Velocity and Compressor Mach Number. NACA TN 1664, 1948.

A Helmholtz resonator with variable volume, which measures the velocity of sound in fluids, is especially adaptable for use in rotating machines. Better than 0.5-percent accuracy is obtained in measuring the compressor Mach number (blade tip speed/inlet stagnation speed of sound).

205. Erwin, John R., and Emery, James C.: New Approach to Axial Compressor Cascade Testing Technique. S.A.E. Quart. Trans., vol. 4, no. 2, Apr. 1950, pp. 275-286.

A new method for boundary-layer removal on tunnel walls of cascades, involving suction through porous walls at low speeds, is explored for a range of Reynolds numbers. The results indicate that porous-wall suction applied to cascades of compressor blades makes possible the attainment of two-dimensional flow effects and for the first time enables reliable cascade data to be obtained. Another conclusion is that conventional compressor blade cascades of aspect ratio 4 do not simulate the two-dimensional case. Evidently very large aspect ratios are required.

206. Hassan, Mahmoud Ali: Anwendungen der elektrolytischen Methode auf die Betz'sche Theorie der Spaltverluste an Schaufelgittern. Mitt. aus dem Inst. f. Aero. (Zurich), Nr. 15, 1949. (Application of the Electrolytic Method to the Betz's Theory of Clearance Losses on Blade Cascades.)

An electrolyte in a tank analogue to potential flow is used to determine the induced tip-clearance losses for rotating blades. The method compares well with an analytic solution for the flow around the walls with induced cross flows.

207. Erwin, John R., and Emery, James C.: Effect of Tunnel Configuration and Testing Technique on Cascade Performance. NACA Rep. 1016, 1951. (Supersedes NACA TN 2028.)

This report is one of a series of investigations into the characteristics of airfoil cascades and the application of cascade data to turbomachine design use.

Among earlier reports on blade design data resulting from this study were surveys 121 and 150. It was recognized early that accurate blade design data were difficult to obtain even in a two-dimensional configuration, because of boundary-layer and viscous flow complications.

The experimental lift coefficients for airfoils in cascades are less than those obtained theoretically (survey 122). Furthermore, the differences are greater than for the corresponding isolated airfoils, and the discrepancies were attributed largely to end-wall effects. This is a reflection on the validity of the assumption of two-dimensional flow as required for the theoretical solution. Even more serious complications may be expected in a three-dimensional configuration typical of a compressor. In survey 151, the investigation was extended into the highly-cambered blade range. At the same time, investigations were finding ways of eliminating the end-wall effects and improving the comparison between experimental and theoretical data. The results obtained in two-dimensional-cascade tests, in which boundary-layer removal was attempted by the use of suction slots and suction through porous end walls, are reported. The investigation was made to determine the influence of aspect ratio, Reynolds number, and boundary-layer control on the performance of airfoils in cascade at low speeds.

Tests with solid walls disclosed large discrepancies between theoretical two-dimensional and experimental data (not for turbine cascades, however). Proper control of the boundary layer by slots and porous walls resulted in flows that compared well experimentally with two-dimensional-cascade flow theory.

The results and techniques of this investigation were later applied in survey 124.

208. Grant, Howard P.: Hot-Wire Measurements of Stall Propagation and Pulsating Flow in an Axial Flow Inducer-Centrifugal Impeller System. Pratt and Whitney Res. Rep. No. 133, June 1951.

An early investigation of the phenomenon of propagating stalls using hot-wire instrumentation and techniques is reported. A full description of the hot-wire anemometer and its uses is presented. How the observed readings from two anemometers, properly spaced, can be interpreted to indicate both the number and velocity of the propagating-stall regions is explained, and examples are presented.

Some of the outstanding characteristics of propagating stalls, such as the number, size, and speed of propagation of the stall regions varying with angle of attack and through-flow velocity, are reported more fully in later investigations (see surveys 61 to 69).

209. Meyer, C. A., and Benedict, R. P.: Instrumentation for Axial-Flow-Compressor Research. Trans. A.S.M.E., vol. 74, no. 8, Nov. 1952, pp. 1327-1331.

The criteria for the use of instruments whose optimum dimensions are based on test data are presented in this report, together with a pertinent bibliography. Temperature measurements by means of thermocouples are discussed, and methods for increasing their accuracy are investigated. Among the other instruments investigated are claw-type, spherical, disk-type, tip-clearance, averaging, and wake finger probes, Newton remote traverses, thermocouples, and static taps. Visualization methods are also discussed.

210. Keast, F. H.: High-Speed Cascade Testing Techniques. Trans. A.S.M.E., vol. 74, no. 5, July 1952, pp. 685-694.

A method for boundary-layer control is investigated by injecting air through slots immediately upstream of a cascade. The design and use of wedge-shaped probes and automatic integrating yaw and total-pressure instruments are discussed. Cascade measured test results are converted into turbomachine performance charts, and the calculations required are performed by punch-card methods.

A discussion by W. B. Briggs on the tunnel characteristics, turbulence levels, and instrumentation of the investigation is likewise presented.

211. Brunot, A. W., and Fulton, R. O.: A Clearanceometer for Determining Blade-Tip Clearances of Axial-Flow Compressors. Trans. A.S.M.E., vol. 75, no. 1, Jan. 1953, pp. 1-6.

The design and use of an instrument for determining the blade tip clearances of axial-flow compressors are described. The development of a probe-type measuring head, the control circuits for operating the head, results from measurements using the instrument, and illustrations showing clearances in axial-flow compressors at various operating conditions are presented.

It is noted that, at the time, the instrument was not yet suitable for flight test use because of its weight and power requirements. Further research designed to simplify and adapt the instrument are in progress.

### SECTION III. END LOSSES AND SECONDARY FLOWS

212. Meldahl, A.: End Losses of Turbine Blades. Jour. Am. Soc. Naval Eng., vol. 54, no. 3, Aug. 1942, pp. 454-466.

Tip-leakage loss calculations, made on the basis of tests of a single reaction turbine, show an approximately linear variation of efficiency with tip clearance. The study discloses that the total end losses depend upon the blade shape at the

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tip, the pitch, the type of flow and turbulence, the Reynolds number, and the Mach number. Blade end losses are considered more than mere clearance phenomena. Secondary-flow effects and vortex generation with and without tip clearance are noted.

213. Ainley, D. G., and Jeffs, R. A.: Analysis of the Air Flow Through Four Stages of Half-Vortex Blading in Axial Compressors. R. & M. No. 2393, British A.R.C., Apr. 1946.

Traverses were made behind every blade row at the operating condition for peak efficiency and pressure rise at low Mach numbers. A steady deterioration of the radial distribution of axial velocity through the compressor was noted. Agreement between the measured velocities and those predicted on the basis of simple radial equilibrium was obtained only at the mean section. As a result of this deterioration, a "work done" factor was necessary to correlate the actual work done per stage with the theoretical value for compressor design. The air deflections showed good agreement with values predicted by cascade tests but departed considerably from design values. This departure is attributed to secondary flows and the changes in velocity profile. Measurements disclosed both the presence of trailing vortices from every blade and secondary flows induced by the relative motion of the blade tips and end wall. It may be desirable to permit the variation of axial-velocity distribution through a compressor in order to reduce the stalling at the roots and tips of high-pressure stages. Blade shrouding is likewise suggested.

214. Hausenblas, H.: Flow in a Four-Stage Axial Compressor. Z.V.D.I., vol. 94, no. 9, 1952, p. 246.

The analysis of the flow through the compressor of survey 213 is discussed, and attention is called to the shift of the losses from inner radial positions to outer radial positions in the flow through the rotor. This effect is attributed in part to the secondary-flow mechanism.

215. Weske, John R.: Experimental Investigation of Velocity Distributions Downstream of Single Duct Bends. NACA TN 1471, 1948.

Velocity and pressure surveys were made at the outlet sections of a large number of duct elbows of varying cross-sectional shapes for a range of Reynolds numbers from  $0.2 \times 10^6$  to  $0.6 \times 10^6$ . The results indicate the same general patterns of flow for all cross-sectional shapes. The radius-ratio variations were significant in regard to their effects on changing these patterns.

Three distinct regions were observed in the common patterns: (1) a core region, equivalent to the main flow region, with fairly constant total pressure throughout and velocity distribution close to potential flow, (2) the layer of peripheral flow, corresponding to the boundary-layer region near the walls, with low axial-velocity components and sizable peripheral velocity components, and (3) a region of eddying flow and low total pressures, in which measurements are difficult to make because of the large yaw values. Region (3) is found near the inner wall of all the curved ducts. When no separation is present, the flow in this region takes on a spiral twisting form in which the total pressure increases with distance from the center of curvature (see secondary-flow reports, surveys 230 to 234).

The displacement of the boundary-layer fluids toward the bend inner wall region was noted, as well as the dependence of the amount of this displacement and the intensity of the spiraling upon the radius of curvature of the bend. The persistence of the spiraling motion for great distances was recorded. As in surveys 230 to 234, neither the length of the inlet duct, nor poor matching of inlet-duct joints and the Reynolds number variations for the range covered, had an appreciable effect on the flow patterns.

216. Weske, J. R.: Secondary Flows in Rotating Blade Passages at High Reynolds Numbers. Proc. Seventh Int. Cong. for Appl. Mech., vol. 2, pt. 1, 1948, pp. 155-163.

A two-dimensional analysis of the radial boundary-layer displacement in a free-vortex main stream away from the blade end region is based on Euler's equation. The analysis includes the effects of blade length and blade angles on the radial-velocity components, and the influence of the radial velocities on the blade surface pressure distribution and on losses in the rotor blade row. The stability of the secondary flows in the boundary-layer regions of the rotor blades is analyzed. A theory is advanced for tip stalling in the rotor blades of an axial-flow compressor.

217. Hausmann, George F.: The Theoretical Induced Deflection Angle in Cascades Having Wall Boundary Layers. Jour. Aero. Sci., vol. 15, no. 11, Nov. 1948, pp. 686-690.

A lifting-line theory is developed for the induced incompressible nonviscous flow in a two-dimensional cascade having wall boundary layers, in order to obtain a general expression for the downwash velocity. The theory is developed for cases of thin boundary layers, in which changes of boundary-layer shape and thickness can be neglected. The velocities induced by a modified vortex system associated with the wall boundary layers of two-dimensional cascades are determined and are considered to approximate corrections of the deflection-angle data for cascade tunnels. Replacing the cascade blades by lifting lines leads to two correction factors, one accounting for the induced deflections due to the trailing vortex system in the wall boundary layers, and the other accounting for the decrease in total circulation of the bound-vortex system.

The effects of blade length, boundary-layer thickness, and the number of blades on the deflection angles of several cascades are shown.

218. Carter, A. D. S.: Three Dimensional Flow Theories for Axial Compressors and Turbines. Rep. No. R.37, British N.G.T.E., Sept. 1948. (See also War Emergency Issue No. 41 pub. by Inst. Mech. Eng. (London). Reprinted in U.S. by A.S.M.E., Apr. 1949, pp. 255-268.)

Secondary flows in cascades are discussed from experimental and theoretical viewpoints. The secondary flows are attributed to trailing-edge vortices and to phenomena associated with both nonuniform approach velocities and spanwise variation of circulation along the blades. Quantitative estimates of the flow deflections are attempted. The flow of a perfect fluid through an annulus is considered, and the flow requirements of radial equilibrium are discussed.

The effects of tip clearance and relative motion between the blades and the wall are discussed in general terms. The boundary-layer displacement, work done, and induced drag are linked together.

219. Carter, A. D. S., and Cohen, Elizabeth M.: Preliminary Investigation into the Three-Dimensional Flow Through a Cascade of Aerofoils. R. & M. No. 2339, British A.R.C., Feb. 1946.

A general discussion is presented of secondary flows in cascades of straight airfoils based on experimental observations of total-pressure, static-pressure, and flow-angle measurements upstream and downstream of cascades. The tunnel wall boundary layers are observed to have a considerable displacement effect on the main flow as a result of their interaction with blade trailing-edge vortices, which are attributed to variation of circulation along the blades (see survey 230).

220. Squire, H. B., and Winter, K. G.: The Secondary Flow in Cascade of Aerofoils in Non-Uniform Stream. Rep. No. AERO. 2317, British R.A.E., Mar. 1949. (See also Jour. Aero. Sci., vol. 18, no. 4, Apr. 1951, pp. 271-277.)

This theoretical and experimental report was the forerunner of a series of studies of nonviscous secondary-flow investigations (e.g., surveys 220 to 228). The purpose of the nonviscous secondary-flow approach is to obtain a good physical picture of the flow characteristics by use of simplified equations. However, precise details of the flow behavior are not obtainable from this approach. The theory considers the secondary flow in a guiding passage (pipe, channel, blade row) resulting from a nonuniform total pressure upstream of the turning configuration; that is, corresponding to the case of a very thick boundary layer. As the fluid turns, secondary flows develop and are attributed to components of vorticity in the same direction as the through-flow velocity vector.

The analyses of this series are based on the following three governing equations for the flow of a steady incompressible nonviscous fluid:

$$\text{curl } (\bar{V} \times \Omega) = 0$$

$$\text{div } \Omega = 0$$

$$\text{div } \bar{V} = 0$$

where  $\bar{V}$  and  $\Omega$  are the velocity and vorticity vectors of the flow, respectively. An expression is derived for the secondary-flow component generating vorticity  $\xi_1$  parallel to the through-flow downstream. The theory assumes the secondary-flow velocities as a perturbation on the potential two-dimensional free-vortex primary flow. From knowledge of the vorticity component  $\xi_1$ , the secondary-flow velocities are found. By addition to the primary flow, a first approximation to the actual flow may be obtained. No self-transport of vorticity is assumed.

The secondary flows are responsible for losses comparable to induced drag losses of the same order of magnitude as the pressure losses resulting from profile drag effects. The agreement of experiment with theory was fairly good, (see surveys 48 and 237).

221. Stephenson, J. M.: Secondary Flow in Cascades. Jour. Aero. Sci., vol. 18, no. 10, Oct. 1951, pp. 699-700.

A simpler alternate derivation of the results of survey 220 is attempted by use of the principle of conservation of circulation in order to calculate the gyroscope effect (noted in survey 220), where flow into bends contain vorticity.

222. Loos, H. G., and Zwaaneveld, S.: Secondary Flow in Cascades. Jour. Aero. Sci., vol. 19, no. 9, Sept. 1952, pp. 646-647.

Discrepancies in the method of adding trailing vortices noted in survey 221 are discussed. Another method is developed using the method of survey 225.

223. Yeh, H.: Secondary Flow in Cascades. Jour. Aero. Sci., vol. 19, no. 4, Apr. 1952, pp. 279-280.

The chief result of survey 220 is obtained for the first approximation to the secondary flows downstream of a cascade with parallel nonuniform spanwise flow entering the cascade.

224. Hawthorne, W. R.: Induced Deflection Angle in Cascades. Jour. Aero. Sci., vol. 16, no. 4, Apr. 1949, p. 252.

An analysis of flow patterns (similar to that of survey 217) leads to the conclusion that the flow deflection in a cascade of finite span is greater than that in a cascade of infinite span.

225. Hawthorne, William R.: Secondary Circulation in Fluid Flow. Gas Turbine Lab., M.I.T., May 1950. (See also Proc. Roy. Soc. (London), ser. A, vol. 206, no. A1086, May 7, 1951, pp. 374-387.)

As in survey 220, secondary flows are attributed to components of vorticity in the same direction as the flow vector. Beginning with the same three governing equations as survey 220, the local rate of change along the streamlines of this secondary-flow vorticity is derived in an analysis assuming an incompressible nonviscous steady main flow with no body forces. The secondary flow is considered a perturbation on the primary flow; no self-transport of vorticity is permitted. An expression is found relating the secondary-flow vorticity component to the velocity, the streamline curvature, and the incoming total-pressure gradient. By integration along the two-dimensional streamlines, the effects of the secondary-flow vorticity distribution are determined. By addition of these perturbation effects with the two-dimensional main-stream flow, the streamline displacements resulting from secondary-flow effects are evaluated.

Applying this theory to the flow in pipe bends results in secondary flows that are oscillatory, and not spiral. This result is highly qualified by the assumptions made in developing the theory. Agreement with experiment indicates that, while fluid friction is responsible for the secondary flows as it generates the varying total pressure of the fluid at the inlet, it is this stagnation-pressure gradient, not the friction losses in the pipe (or channel) bend itself, which governs the secondary-flow development throughout the fluid, for this particular investigation.

226. Kronauer, Richard E.: Secondary Flows in Fluid Dynamics. Pratt and Whitney Res. Rep. No. 132, Gordon McKay Lab., Harvard Univ., Apr. 1951.

To obtain more exact solutions for the nonviscous secondary-flow analyses of surveys 225 and 220, an iteration process of successive approximations is developed. Starting with an assumed first approximation to the velocity field under investigation, a vortex field is calculated by means of the Cauchy vorticity equations. The vortex field is then integrated to give a new velocity field, and the process is repeated until discrepancies between successively calculated velocity fields are small.

The computational difficulties make it undesirable to calculate more than one step in the successive approximation scheme. In order for the results to be meaningful, a good starting approximation must be made. The author believes that enough is usually known about the desired solution to do this.

227. Eichenberger, Hans P.: Shear Flow in Bends. Tech. Rep. No. 2, Office Naval Res., Gas Turbine Lab., M.I.T., Apr. 15, 1952. (Contract N5ori07848.)

Even though the nonviscous-flow analyses (surveys 220, 225, and 226) are simplified approaches to the secondary-flow problem, solutions of the governing equations are nonetheless difficult to obtain. Use of more simplifying assumptions would qualify still further the validity and meaningfulness of the results so obtained. A more exact perturbation-type solution involving series expansions with Bessel functions, developed in this report, provides a partial answer to the question. The simplified solutions for the cases computed agree well with the more exact solution.

228. Ehrich, F. F., and Detra, R. W.: Transport of the Boundary Layer in Secondary Flow. Jour. Aero. Sci., vol. 21, no. 2, Feb. 1954, pp. 136-138.

It is assumed that the secondary velocities, and thus the perturbation stream function (representing the secondary flows) in the plane perpendicular to the through-flow, build up linearly through the cascade. On this basis, a stepwise procedure is developed for calculating the movement of a particle of fluid in the plane perpendicular to the main-flow direction as it progresses through the passage. Thus, both the streamline paths and the secondary vorticity can be determined through a bend.

229. Silberman, E.: Importance of Secondary Flow in Guide Vanes Bends. Proc. Third Midwestern Conf. on Fluid Mech., Univ. Minn., 1953, pp. 669-686.

An experimental investigation of the secondary flows in 90° guide-vane bends is presented for three vane profiles and a range of solidities. The major losses due to secondary flows in cascade rows occurred downstream. These losses are occasioned by the increased wall shear resulting from transport, by secondary-flow action, of high-velocity main-stream fluid onto the duct walls downstream of the vanes. The generally small effects of secondary flows on main-stream deflection are pointed out, but the locally important secondary-flow effects on deflection angles near the end walls are ignored.

The effect of secondary flow on total-pressure loss is divided into two parts: (1) loss that can be measured immediately behind the vanes, and (2) the increased shear-flow loss previously discussed, which is measured farther downstream.

230. Herzig, Howard Z., Hansen, Arthur G., and Costello, George R.: A Visualization Study of Secondary Flows in Cascades. NACA Rep. 1163, 1954. (Supersedes NACA TN 2947.)

The secondary-flow patterns in the boundary layers of various cascades and turbomachine configurations were investigated for a range of flow speeds from about 30 feet per second to Mach 1.4. The general plan adopted was to investigate secondary flows in simple two-dimensional configurations first, and then to proceed to more general three-dimensional configurations. Flow-visualization techniques, such as smoke filaments, were used to trace the patterns at low speeds. Chemical and paint traces were employed at the higher speeds. The information obtained from such boundary-layer tracing was correlated with total- and static-pressure and flow-angle surveys made by more conventional instrumentation, in an effort to obtain an over-all picture of the fundamental flow patterns that exist in real flows.

The end-wall boundary layer of a two-dimensional cascade undergoes complicated three-dimensional motions as it moves under the influence of main-stream turning. Overturning in the boundary layer results in cross-channel secondary flows that tend to accumulate near the suction side of the passage, giving rise to flow vortex formation well up within each cascade passage. The size and tightness of the vortex generated depend upon the main-flow turning in the cascade. In downstream blade rows, such a flow vortex can cause serious flow disturbances and unfavorable flow angles. Thus, while very little energy may be involved in vortex formation, the losses instigated by the flow vortex in succeeding stages may be important.

Two major tip-clearance effects were observed: the formation of a tip-clearance vortex, and the scraping effect of a blade in relative motion past the wall boundary layer. The magnitudes of these effects are functions of the tip speed, the amount of tip clearance, the blade-tip shape and stagger, the main-flow speed and turning, and the direction of relative motion. Detailed studies of the tip-clearance flows



and the blade surface boundary-layer flows indicate possible methods for improving the blade-tip loading characteristics of compressors and turbines. By careful choice of the optimum tip-clearance size (not necessarily the minimum clearance), blade-end shape and speed, main-flow turning, and so forth, the tip-clearance-region disturbances can be minimized.

Extensions and applications of both the information presented in this report and the experimental research techniques employed are made in succeeding investigations (see surveys 231 to 233).

231. Rohlik, Harold E., Kofskey, Milton G., Allen, Hubert W., and Herzig, Howard Z.: Secondary Flows and Boundary-Layer Accumulations in Turbine Nozzles. NACA Rep. 1168, 1954. (Supersedes NACA TN's 2871, 2909, and 2989.)

An investigation of secondary-flow loss patterns originating in three sets of turbine-nozzle blade passages was conducted by means of flow-visualization studies and detailed flow measurements. For all cases, high loss values were measured in the fluid downstream of the corners formed by the suction surfaces of the blades and the shrouds, and these losses were accompanied by discharge-angle deviations from design values. Despite the sizes of the loss regions and the angle gradients, overall mass-averaged blade efficiencies were of the order of 0.99 and 0.98, indicating that such mass-averaged blade efficiencies do not convey much information about the flow conditions and are not a good index of blade performance.

The inner-wall loss core associated with a blade of a turbine-nozzle cascade is largely the accumulation of low-momentum fluids originating elsewhere in the cascade. This accumulation is effected by the secondary-flow mechanism, which transports the low-momentum fluids radially in the blade wakes and boundary layers and across the channels on the walls. At one flow condition investigated, the radial transport of low-momentum fluid in the blade wake and on the suction surface near the trailing edge accounted for approximately 65 percent of the inner-wall loss core, about 30 percent resulting from flow in the thickened boundary layer on the suction surface, and about 35 percent from flow in the blade wake.

The degree to which blade-surface velocity profiles affect the magnitude and concentration of loss cores was investigated by comparing three nozzle-blade configurations. Flow-visualization studies and flow measurements at the lower Mach numbers indicate that thickened blade boundary layers existing on the blades near the outer shroud, as a result of unfavorable blade surface velocity profiles, may provide the conditions required for passage vortex formation. Under these conditions, sizable outer-shroud loss cores are found at the nozzle discharges. Blades having thinner, two-dimensional suction profile boundary layers, however, appear to offer resistance to passage vortex formation near the outer shroud, and an inward radial flow of low-momentum air results in the blade wake. Under these conditions, the inner-shroud loss region at the nozzle discharge is large, while the outer-shroud loss region may be quite small in comparison.

In both cases, reduced loss accumulations along the outer shroud are obtained at the higher Mach number, as shock boundary-layer thickening on the blade surface provides an additional path for the radially inward flow of low-momentum fluid. Therefore, the results indicate that passage vortices may not form for all blade configurations and flow conditions but their formation may be governed to a large extent by blade boundary-layer thickness and separation. Comparison of well-designed constant-discharge-angle and free-vortex type blades indicates that the secondary-flow loss differences for these blades are so small that the choice of the type of blading, based solely on secondary flows, is of negligible concern.

232. Herzig, H. Z., and Hansen, A. G.: Visualization Studies of Secondary Flows with Applications to Turbomachines. Preprint No. 53-A-56, A.S.M.E., 1953.

Most of the factual information obtained in surveys 230 and 231 is presented in condensed form, and methods for control and diversion of the secondary flows for application to turbomachinery are discussed briefly. Centrifugal effects were not included in this study.

233. Kofskey, Milton G., and Allen, Hubert W.: Smoke Study of Nozzle Flows in a Low-Speed Turbine. NACA TN 3260, 1954.

Still and motion pictures were made of boundary-layer and wake secondary-flow phenomena, visualized by smoke in a very low-speed turbine specially designed for the purpose. The motion pictures effectively present many of the phenomena described by photographs and works in the earlier reports in the series (surveys 230 to 232). The boundary-layer cross-channel deflections, radial flows, accumulations, and vortex formation near the suction surfaces are all demonstrated. The double-layer nature of the cross-channel boundary-layer flow is discussed, and blade-row interference phenomena are demonstrated. The motion of a downstream rotor blade row is seen in the motion pictures to produce pulsations in the nozzle trailing-edge radial-flow patterns.

The motion-picture supplement may be obtained on loan from NACA Headquarters, Washington, D. C.

234. Stanitz, John D., Osborn, Walter M., and Mizisin, John: An Experimental Investigation of Secondary Flow in an Accelerating, Rectangular Elbow with  $90^\circ$  of Turning. NACA TN 3015, 1953.

Total-pressure surveys at the inlet and exit and the static-pressure distribution were measured on the pressure and suction surface of a  $90^\circ$  elbow designed for accelerating flow so that boundary-layer separation was avoided (Stanitz, John D.: Design of Two-Dimensional Channels with Prescribed Velocity Distributions Along the Channel Walls. I - Relaxation Solutions. NACA TN 2593, 1952). The data, which were analyzed using the continuity and momentum laws, disclosed the presence of flow vortices near the suction surfaces of the elbow. The vortex showed no appreciable spanwise motion as it moved downstream from the elbow exit.

Spoilers at the inlet were used to generate a range of six boundary-layer thicknesses on the plane walls of the elbow. As the spoiler size and the boundary-layer thickness increased, the vortex changed in size and spanwise position, these changes being quite sudden in a certain part of the boundary-layer-thickness range. The report speculates that at this point, the boundary-layer thickness has become such that the viscous effects have reduced importance (as in survey 230). The analysis suggests that the strength of the secondary vortices and the energy of the secondary flows are small.

235. Smith, Leroy H., Jr.: Three-Dimensional Flow in Axial-Flow Turbomachinery. Pt. I. Theoretical Determination of Secondary Flow. Rep. 1-14, Mech. Eng. Dept. Internal Flow Res., The Johns Hopkins Univ., Nov. 1953. (Air Force Contract AF-33(616)-152.)

Assuming nonviscous flows, the direct and inverse axisymmetric flow solutions are presented in a form intended to expedite secondary-flow discussion. The assumptions and limitations on the methods are pointed out. The induced turning and radial velocities due to secondary-flow effects are deduced from the axisymmetric solution.

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The necessary conditions for significant secondary flows are expressed, and methods for considering their effects in compressor design are investigated. Trailing vortex sheets, their effects, and secondary vorticity associated with circulation downstream of a blade row are discussed.

236. Smith, Leroy H., Jr.: Three-Dimensional Flow in Axial-Flow Turbomachinery. Pt. II. Experimental Investigations. (Air Force Contract AF-33(616)-152.) The Johns Hopkins Univ., May 1954.

A single-stage low Mach number compressor was built according to the theory of the first part of the investigation (see survey 235), in order to determine the feasibility of using highly rotational flows to achieve high pressure ratios per stage with high mass flows. The test results indicate that it is possible to use highly rotational flows, that the design camber can be made for such flows by axisymmetric theory with good results, and that compressors with good efficiency can thereby be achieved.

A good estimate of the magnitude of the expected secondary flows can be obtained from the axisymmetric solution. For long passages and conditions where there are large secondary flows, spanwise energy transport may make the axisymmetric solutions inaccurate (see survey 231). For such cases, a modified axisymmetric solution would be required. The effects of blade-thickness taper may be significant and should be considered (see survey 142). The effects of vorticity self-transport causing passage vortex formation (see survey 230) and the unsuitability of the two-dimensional continuity criterion for analyzing the behavior of thin wall boundary layers are discussed.

237. Ehrich, F. F.: Secondary Flows in Cascades of Twisted Blades. Preprint No. 432, Inst. Aero. Sci., Jan. 1954.

The flow of a nonviscous fluid with spanwise nonuniform velocity distribution through a two-dimensional cascade with nontwisted blades is discussed in survey 220, the uniform flow through a cascade of twisted blades is reported in survey 48. In the present report nonuniform flows past twisted blades are analyzed, the secondary flow being considered as a perturbation on the main flow. The general result obtained reduces exactly to the solution of survey 220, and is qualitatively and quantitatively similar to the results of survey 48, for their respective cases.

238. Dean, Robert C., Jr.: Secondary Flow in Axial Compressors. Gas Turbine Lab., M.I.T., May 1954.

A review and evaluation of a great deal of secondary-flow research, both theoretical and experimental, is presented, and a bibliography of over 40 related reports is surveyed. Discussions of the assumptions, the methods employed, and the interpretations of the results obtained are included. The net result is portrayal of the physical phenomena involved, as well as an appreciation for the problems incurred in handling and investigating secondary flows.

A theoretical treatment of the problem of determining the secondary-vorticity behavior of the boundary layer skewing through a cascade is developed, tested, and evaluated in a manner emphasizing the governing physical parameters. One result indicates that the strength of a vortex formed in a passage with large main-flow turning may be expected to increase as the inlet wall boundary becomes thinner or the turning angle larger.

An approximate theory is developed to include the effects of streamwise pressure gradients, always previously ignored. The analysis applies to thin boundary layers for cases of small main-stream turnings but does not apply to the behavior of

boundary-layer fluids close to the walls in diffusing passages. The experimental verification of the analysis, which was difficult and not quite suitable, but hopeful, indicates the importance of considering pressure stresses in secondary-flow analyses. To widen the scope of the analysis, the effects of shear stresses are investigated by means of a momentum analysis of a thin laminar boundary layer on the plane wall of a turning passage. The approximate analysis indicates the importance of viscous stresses (and turbulence stresses) on the boundary-layer flows near the wall.

239. Mellor, George L., Jr.: Theoretical Consideration of Secondary Flow. Rep. 27-2, Gas Turbine Lab., M.I.T., Dec. 1954. (Contract N5-ori-07848.)

A more exact extension is made here of the analysis presented by survey 235, which investigates the laws of vorticity transport for incompressible nonviscous steady flows. The main or primary flow is obtained by an axisymmetric solution, and the secondary flows are assumed to be small perturbations on the primary flow. Self-transport of vorticity is ignored. Mixing losses in the main stream occasioned by secondary flows may be significant.

240. Wu, Chung-Hua, and Wu, Wen: Analysis of Tip-Clearance Flow in Turbomachines. Tech. Rep. No. 1, Dept. Mech. Eng., Gas Turbine Lab., Polytechnic Inst. of Brooklyn, July 1954. (Contract Nonr 839(04) NR-062-172, Office Naval Res., U.S. Navy.)

A basic analysis is presented of the blade tip-clearance flow in turbomachinery. The flow is assumed viscous, incompressible, and steady. Both low and high Reynolds numbers are considered.

An analytical solution of the low Reynolds number case results in an expression for the velocity distribution in the tip-clearance space in terms of pressure gradient and relative blade-to-wall speed. No analytical solution can be obtained for the high Reynolds number case. An approximate solution yields an expression similar to that obtained for low Reynolds numbers.

The pressure gradient over a blade section at the tip is affected by the blade loading, the shape of the tip section, the casing boundary-layer characteristics, the viscosity, and the relative blade-to-casing motion (see surveys 212 and 230). Velocities through the tip-clearance space are governed by the pressure gradient, which varies as the square of the tip clearance. Mass flow through the tip-clearance region depends upon the velocity distribution and the pressure gradient, and varies with the cube of the tip clearance.

#### SECTION IV. GENERAL HISTORICAL INTEREST

241. Spannhake, W. (L. J. Goodlet, trans.): The Three-Dimensional Theory of Turbines and Pumps for Incompressible Fluids. R.T.P. Trans. No. 1568, British M.A.P. (From Forsch. Geb. Ing.-Wes., vol. 8, no. 1, Jan-Feb. 1937, pp. 29-34.)

The flow through diffusers and impellers is examined by means of bound vortices and the infinite number of blades theory. Using source-sink representations, the interaction of blades, casing walls, and disks is proved and discussed. The series representation convergence is slow (see survey 244).

242. Eckert, B.: Possibilities of Reducing the Length of Axial Superchargers for Aircraft Motors. NACA TM 1132, 1947.

Several types of construction, which might permit sufficient reduction of the length of axial blowers for possible use in aircraft, are described: (1) a three-stage blower with adjustable rotor blades; (2) a three-stage axial blower with guide vanes; (3) a four-stage axial supercharger with counterrotating rotors, gearing, and intermediate guide vanes; (4) a four-stage axial supercharger with counterrotating rotors and two gearings; (5) a five-stage supercharger similar to the last; and (6) a five-stage counterrotating axial blower with outside rotating drum. Comparisons of the various types are tabulated.

243. Sørensen, E.: Constant-Pressure Blowers. NACA TM 927, 1940.

The advantages of constant pressure design are discussed. All the energy imparted by the rotor to the fluid is in the form of velocity head. Reduction in axial-flow area provides the rise in axial-flow velocity, which in turn causes the work input to appear as flow kinetic energy rather than as a pressure rise.

Cavitation and separation hazards are great, and the diffuser design becomes critical. The chief advantage is in cheap construction.

244. Sorensen, E.: Potential Flow Through Centrifugal Pumps and Turbines. NACA TM 973, 1941.

The two-dimensional complex potential for the flow through blades at rest and with rotation is given as definite integrals, as compared with the series representation of survey 241. This avoids the difficulties inherent in the slow convergence of the series.

The performance is computed and comparisons are made with the infinite number of blades method obtained by Euler's formula. The effects of variable numbers of blades are discussed, and sample calculations are provided.

An analysis and computation for a flow potential due to blade rotation is presented.

245. Ackeret, J.: Zum Entwurf dichtstehender Schaufelgitter. Schweiz. Bunztg., Bd. 120, Nr. 9, Aug. 29, 1942, pp. 103-108.

The two-dimensional potential flow past a finite number of blades is obtained by means of a source-vortex representation.

246. Betz, A.: Axial Superchargers. NACA TM 1073, 1944.

The great length and narrow range of good performance of axial superchargers are discussed. An analysis discloses that the performance range disadvantages of axial compressors are not as large as they first appear. Methods of decreasing the length are discussed. Included are general qualitative discussions concerning improved flow about the blades, use of shorter guide vanes, counterrotating impellers, and increased stage performance by better design, manufacture, and use of supersonic velocities.

247. Howell, A. R.: The Present Basis of Axial Flow Compressor Design. Pt. I. - Cascade Theory and Performance. R. & M. No. 2095, British A.R.C., June 1942.

General discussions are presented of vortex-flow theory and design, test analyses, Mach number corrections, tip-clearance efficiency correction, and three-dimensional flows in compressors.

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CP-11 back

Experimentally, the variation of profile loss with Reynolds number discloses that inlet Reynolds numbers below  $3 \times 10^5$  are critical. Low-speed cascade tests results are contained in two curves of Mach number effects on the stalling deflection of the air and the efficiency.

248. Howell, A. R.: Design of Axial Compressors. War Emergency Issue No. 12 pub. by Inst. Mech. Eng. (London), 1945. (Reprinted in U.S. by A.S.M.E., Jan. 1947, pp. 452-462.)

The use of basic fluid dynamics in compressor design is explained, as well as the bases and use of some simplified design methods and the final check of a design by performance estimation. Main compressor types, such as reaction or impulse compressors, generalized design curves, and flow coefficients are discussed.

249. Weinig, F.: Basic Considerations in the Design of Axial Flow Compressors. Pt. A - Calculation Fundamentals for Multi-Stage Axial Compressors. BUSHIPS 338, vol. 5, May 1946.

To combine single stages successfully into a multistage compressor having the desired output, reliable preliminary calculations must be made. In order to achieve this purpose, basic concepts are outlined for single-stage and multistage compressors including several efficiencies, types and sources of loss, velocity triangles, throttling, lift, drag, and circulation coefficients (blade loading), and others. A brief discussion is presented of Reynolds and Mach number effects, stage-matching requirements, and the influence of diameter ratio on efficiency. Nomograms that facilitate the calculations are described.

250. Eckert, B., and Weinig, F.: Basic Considerations in the Design of Axial Flow Compressors. Pt. B - Axial Compressors. BUSHIPS 338, vol. 5, May 1946.

Fundamental calculations for the design of axial-flow compressors are given, basic on air-flow and airfoil theory. Methods for increasing the pressure rise per stage, such as higher wheel speeds and careful manufacture, are suggested. The efficiencies of impulse and reaction compressors, the effect of hub-tip radius ratio on over-all efficiency, and the regulation of a compressor are discussed.

The conclusion is reached that axial-flow compressors are better than radial compressors because of their higher efficiency, more air-flow delivery, and better space requirements, that is, smaller frontal area.

251. Encke, W.: Investigations on Experimental Impellers for Axial Blowers. NACA TM 1123, 1947.

Experimental impellers were designed to investigate the possibilities of increasing the pressure rise per stage enough to make multistage axial-flow compressors feasible for aircraft use. A qualitative discussion is presented about the relations among rotor speed, air-handling capacity, pressure rise, efficiency, and lift coefficient.

252. Spannhake, W.: The Problem of the Flow Through Turbomachines. Proc. Midwest Conf. Fluid Dynamics, May 1950.

A general description is presented of some methods of improving turbine and compressor efficiencies by considering the blade circulation and the boundary layer, and by applying conformal mapping.

253. Baxter, A. D.: A Comparison of Axial and Centrifugal Compressor Gas Turbines - An Objective Examination of the Pros and Cons of a Much Debated Subject. Aircraft Eng., vol. XXIV, no. 281, July 1952, pp. 186-188; 197.

The comparisons lead to a chart of values as follows:

			Axial-flow	Centrifugal
Performance	Technical	Efficiency checks:		
		Pressure ratio	7	4.5
		Specific thrust	66.0	56.0
		Specific fuel consumption	.85	1.0
		Dimensions:		
		Weight/thrust	0.3	0.3
		Thrust/frontal area	1000	400
	Operational	Static:		
		Installation	Better	Better
		Maintenance		
		Running:		
Production	Manufacture and cost	Vulnerability		Better
		Flexibility		Better
		Materials		Better
		Machines		Better
		Man-hours		Better

Lewis Flight Propulsion Laboratory  
National Advisory Committee for Aeronautics  
Cleveland, Ohio, August 30, 1955

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